Bailout Stigma*

Yeon-Koo Che[1] Chongwoo Choe[2] Keeyoung Rhee[3]

[1]Columbia University

[2]Monash University

[3]Korea Development Institute

August 18, 2016

*The views expressed in this paper are those of the authors and should not represent those of Korea Development Institute.

Motivation

- Bailout of distressed financial firms is controversial.
  - Bailout involves transfer of taxpayer money and moral hazard.
  - It may be crucial part of the recovery from large scale crises.
- Some literature shows how bailouts can facilitate bank lending in various forms.
  - Government purchase of assets *(Tirole, 2012)*.
  - Debt guarantees by government *(Philippon and Skreta, 2012)*.
- But, bailout-led recovery was slower than anticipated in the crisis 2007-2009.
  - Numerous programs did little to stimulate (inter-)bank lending. Figure
- One reason is the "stigma" attached to the recipients of bailouts:
  - Funding via public program was deemed as a signal of financial weakness.

Some Evidence of Bailout Stigma

- Refusal to accept: Ford, unlike GM and Chrysler, refused rescue loan, saying:

  "Legitimately portraying itself as the healthiest of Detroit's automakers."

- Desire for exit: Signature Bank of New York, the first to repay, stated:

  "We don't want to be touched by the stigma attached to firms that had taken money."

- Forced take-up: The largest banks in U.S. were almost compelled to join TARP.

- Reluctance to use public liquidity program:
  - Banks preferred TAF to Discount Window *(Armantier et al. 2011)*.
  - Financially weak dealers tended to use PDCF *(Krishnamarthy et al. 2014)*.

What We Do

In dynamic adverse selection framework, we analyze

- Effects on Bailouts: How stigma influences effectiveness of bailouts in terms of
  - participation by distressed firms
  - rejuvenation of private lending markets
  - recovery of real economy via initiating socially valuable projects
- Optimal Policy Design:
  - level of bailout ($=$ size of government asset purchase in our model)
  - early rejuvenation of private lending markets (to the extent possible)
  - secrecy vs. transparency

What We Find

- Effects on Bailouts:
  - participation in bailout is lackluster due to attached stigma.
  - bailout terms must contain a stigma premium.
  - bailout indirectly boosts trade by giving reputation-building opportunity.
  - endogenous severity of stigma leads to multiple equilibria.
  - early activated market reduces effects of bailout.
- Optimal Policy Design:
  - with the same market option, secrecy welfare-dominates transparency.
  - immediate market revival is welfare-detrimental.
  - secret bailout without early market revival achieves (constrained) efficiency.

Related Literature

1. Empirical evidence of banks' reluctance to participate in bailouts:
   - Armantier et al. (2011), Cassola et al. (2013), Gauthier et al. (2015), Krishnamarthy et al. (2014), Peristiani (1998), Furfine (2001, 2003)
2. Implentation of optimal intervention in distressed financial market:
   - Philippon and Skreta (2012), Philippon and Schnabl (2013), Tirole (2012)
3. Banks' reputational concerns in secondary markets:
   - Chari et al. (2014), Ennis and Weinberg (2013), La'O (2014)
4. Dynamic adverse selection problems:
   - Daley and Green (2012), Fuchs et al. (2012), Fuchs and Skrzypacz (2015), Hörner and Vielle (2009), Kim (2012)

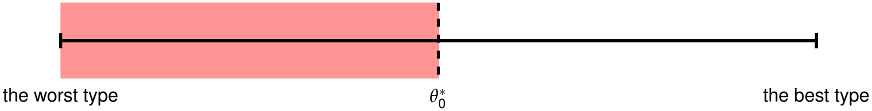
The Simple Economics of Bailout: Tirole (2012)

In a static economy,

- Firm owns 1 unit of legacy asset with value $\theta \in (0, 1)$.
  - Firm privately knows θ.
- Firm can run a project that generates a surplus R at cost I.
  - The net surplus is $S := R - I > 0$.
- Firm should fund I by selling its asset.
  - Buyers at market compete for the asset *a lá Bertrand*.
- Market for asset suffers from adverse selection; may freeze partially or fully.
- Gov't may offer to buy at price p_g prior to the buyers' offer stage.
  - This process can improve asset trade at market *(dregs skimming role)*.
  - But, no stigma effect is captured.

Equilibrium without Government Intervention

$\theta_0^* = p_0^* + S = \mathbb{E}[\theta | \theta \leq \theta_0^*] + S$

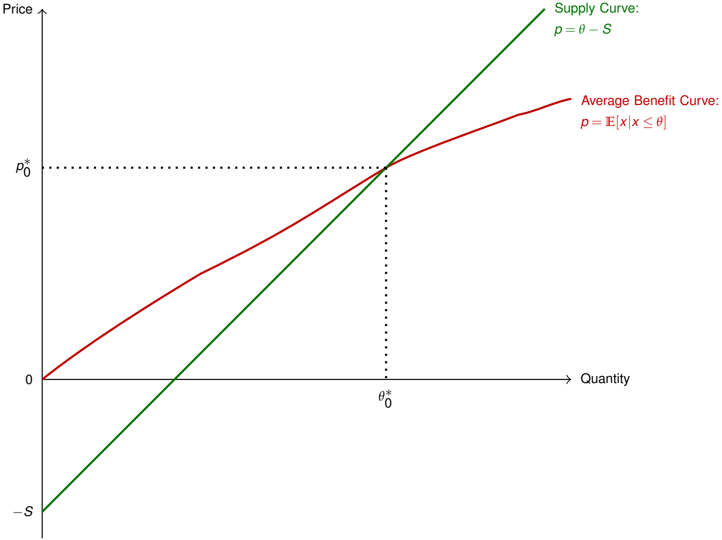


Equilibrium with Government Intervention

$\theta \leq \theta_g$ sell to Gov't

$\hat{\theta}_0 = \hat{p}_0 + S = \mathbb{E}[\theta | \theta \in [\theta_g, \hat{\theta}_0]] + S$

The Simple Economics of Bailout: Tirole (2012)



Model: Two-Period Extension of Tirole (2012)

In a two-period economy ($t = 1, 2$),

Firm owns 2 units of legacy asset with the same value $\theta \in (0, 1)$.

- Firm privately knows θ.
- At $t = 1, 2$, Firm can run new project for net surplus S at cost I.
    - Firm should fund I by selling one unit of its asset in each $t = 1, 2$.

Gov't offers to buy asset at p_g only in $t = 1$.

- Gov't makes the offer before the private buyers do.
- Note: Short-term bailout policy & No nationalization.

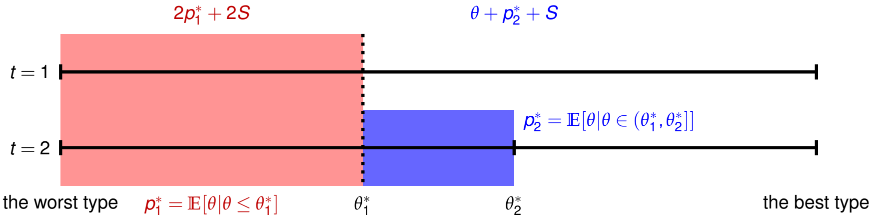
Short-lived buyers in $t = 1, 2$ compete for assets *a lá Bertrand*.

- Adverse selection may lead to partial or full market freeze in each period.

Baseline: Equilibrium without Bailout

The equilibrium has a cutoff structure:

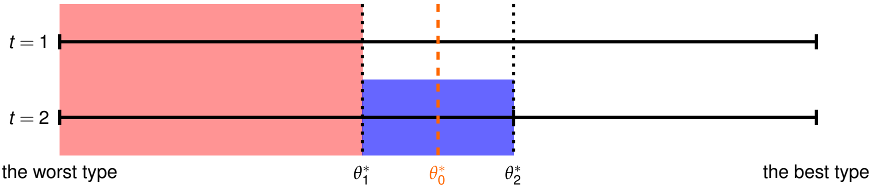
- Lower types begin trading at $t = 1$.
- Higher types hold out in $t = 1$ may sell in $t = 2$.



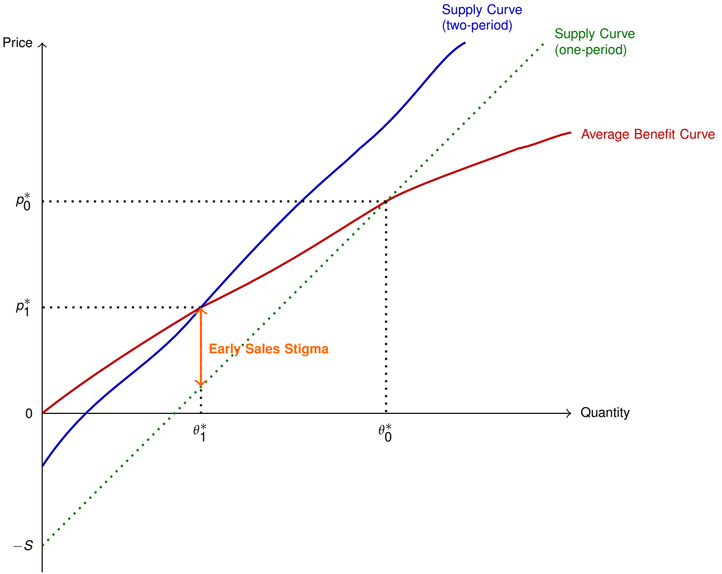
Baseline: Equilibrium without Bailout

Two types of Adverse Selection:

- Static Adverse Selection: High types do not sell *(Akerlof, 1970)*
- Dynamic Adverse Selection: Only low types sell in $t = 1$ (Early Sale Stigma).



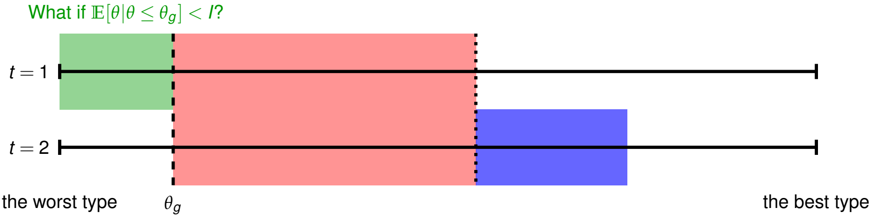
Baseline: Equilibrium without Bailout



Equilibria under Bailout

The influences of bailout differ from *Tirole (2012)*.

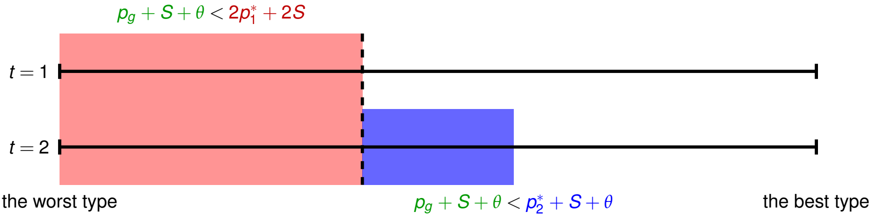
- In static model, Gov't can boost overall trade by buying the lowest types.
  - But, no dynamic consideration of future fundability of the recipients.
- We characterize all equilibria *in the ascending order of bailout terms p_g*.



Equilibria under Bailout: No Impact

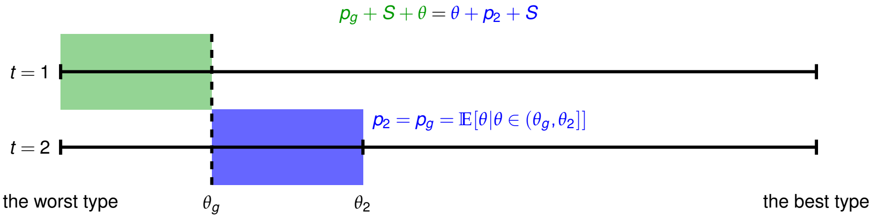
Bailout with offer p_g may have no impact at all.

- If Firm accepts p_g, it is perceived as $\theta = 0$ *off the path*.
- This equilibrium arises even for high bailout terms ($p_g > p_1^*$).



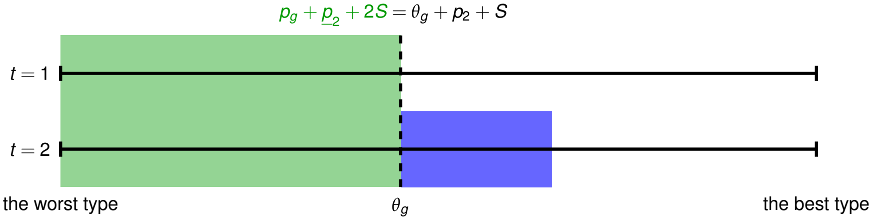
Equilibria under Bailout: No Market Revival/Severe Stigma

- Severe stigma means the recipients cannot sell in $t = 2$ at price $\geq I$.
  - This reduces incentive to sell to Gov't in $t = 1$ (especially for high θ);
  - validates the market belief in $t = 2$, leading to low price $< I$ for recipients.
- Bailout can yield higher trade but indirectly — through private market in $t = 2$:
  - Firm can improve perception by refusing bailout.
- Could lead to a lower trade than without bailout, especially for high S.
  - To avoid this, p_g must be extraordinarily high.



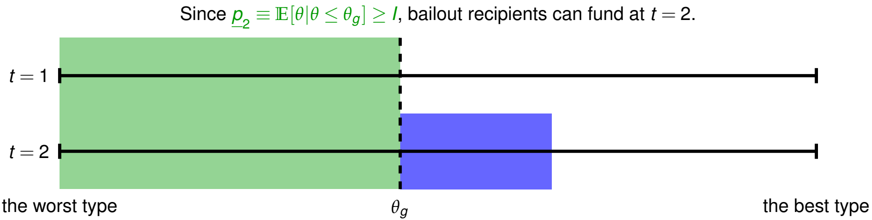
Equilibria under Bailout: No Market Revival/Moderate Stigma

- Moderate stigma means the recipients can sell in $t = 2$ at price $\geq I$.
  - The recipients can sell at price $\geq I$;
  - makes bailout a more attractive option to the firms;
  - yields participation by higher types.
- The severe stigma is of self-fulling nature.
  - We may also have an equilibrium with more moderate stigma.



Equilibria under Bailout: No Market Revival/Moderate Stigma

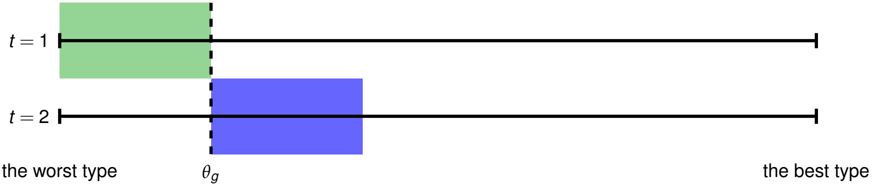
- Moderate stigma means the recipients can sell in $t = 2$ at price $\geq I$.
  - The recipients can sell at price $\geq I$;
  - makes bailout a more attractive option to the firms;
  - yields participation by higher types.
- The severe stigma is of self-fulling nature.
  - We may also have an equilibrium with more moderate stigma.



Equilibria under Bailout: No Market Revival/Moderate Stigma

- Moderate stigma means the recipients can sell in $t = 2$ at price $\geq I$.
  - The recipients can sell at price $\geq I$;
  - makes bailout a more attractive option to the firms;
  - yields participation by higher types.
- The severe stigma is of self-fulling nature.
  - We may also have an equilibrium with more moderate stigma.

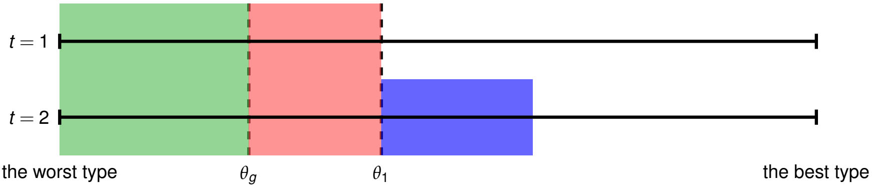
The same p_g may induce bailout equilibrium with severe stigma.



Bailout may immediately rejuvenate market in $t = 1$.

- This equilibrium may coexist with that without early market revival.
- This equilibrium yields lower trading than without market revival at $t = 1$.

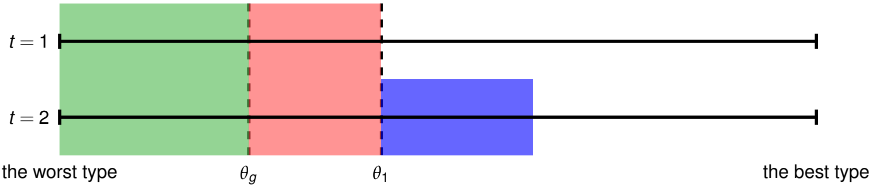
$p_g + \underline{p}_2 + 2S = 2p_1 + 2S = \theta_1 + p_2 + S$



Bailout may immediately rejuvenate market in $t = 1$.

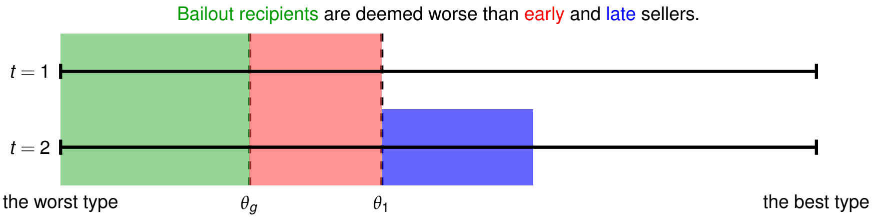
- This equilibrium may coexist with that without early market revival.
- This equilibrium yields lower trading than without market revival at $t = 1$.

Early market revival imposes negative externalities on bailout recipients.



Bailout may immediately rejuvenate market in $t = 1$.

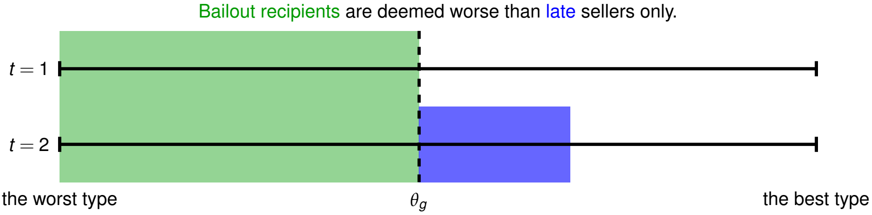
- This equilibrium may coexist with that without early market revival.
- This equilibrium yields lower trading than without market revival at $t = 1$.



Equilibria under Bailout: Market Revival

Bailout may immediately rejuvenate market in $t = 1$.

- This equilibrium may coexist with that without early market revival.
- This equilibrium yields lower trading than without market revival at $t = 1$.



Implications

1. Participation in bailout is lackluster, compared to *Tirole (2012)*.
   - More attractive terms than in static model are required to boost trade.
2. Multiple equilibria with various degrees of stigma.
   - Bailout stigma has self-fulfilling nature.
   - Bailout has discontinuous effects in offer terms p_g.
3. Despite stigma, bailout can create indirect benefits.
   - Firm can improve perception by refusal of bailout.
4. Early market revival dampens overall trading.
   - Availability of market option in $t = 1$ imposes negative *belief* externalities.

Figure: Trading Volume in Alternative Equilibria

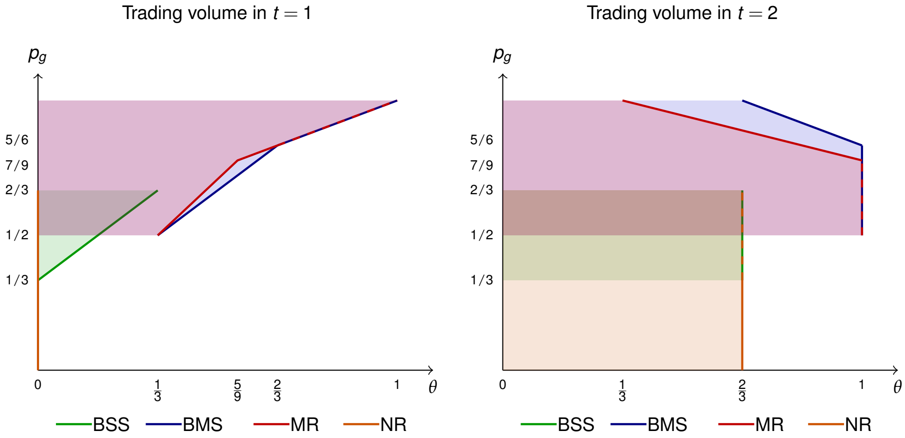
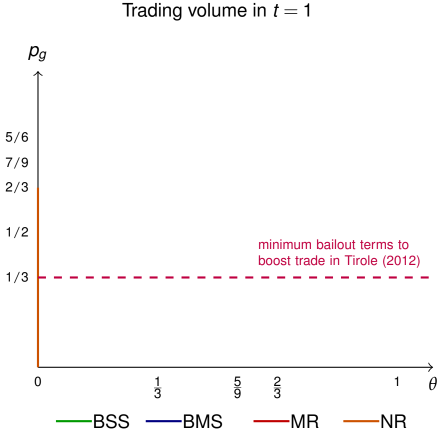


Figure: Trading Volume in Alternative Equilibria



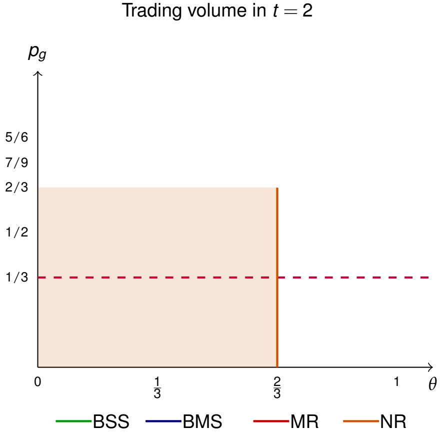


Figure: Trading Volume in Alternative Equilibria

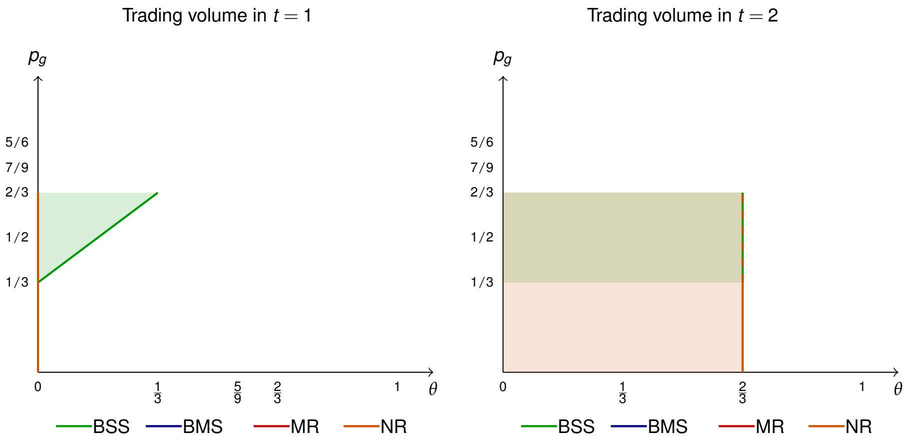


Figure: Trading Volume in Alternative Equilibria

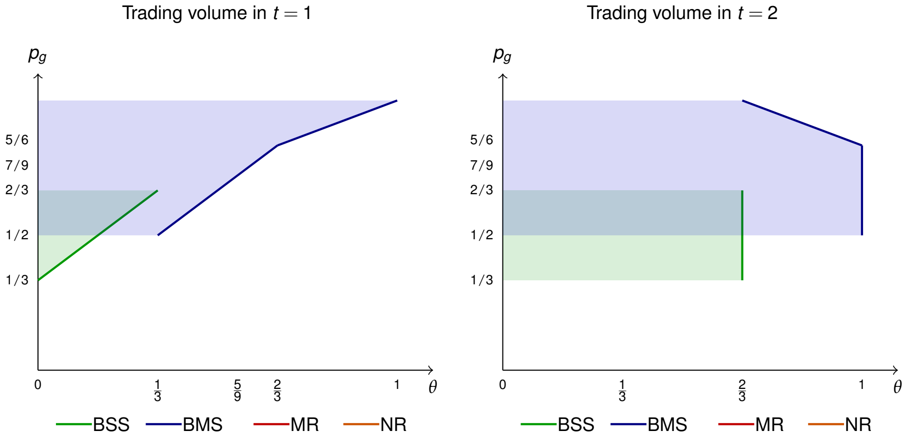
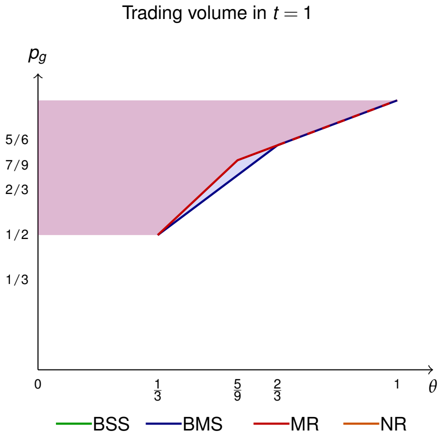
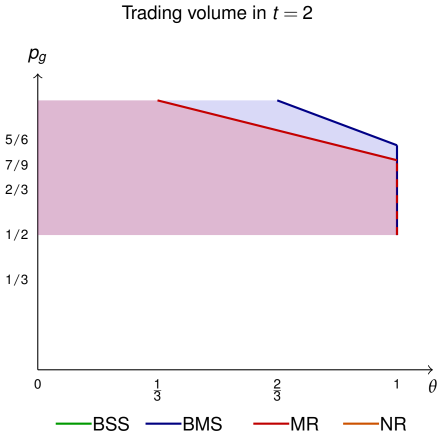


Figure: Trading Volume in Alternative Equilibria



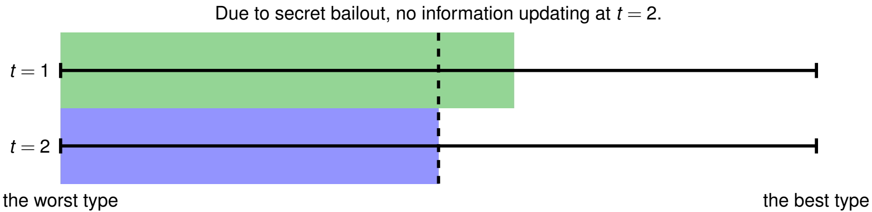


Secret Bailouts

- Policymakers ran liquidity program to alleviate bailout stigma.
  - Banks were reluctant to use liquidity programs by fear of stigma *(FCIR)*.
  - The Fed's Term Auction Facility (TAF) hid borrowers' identity.
    - In uniform-price auction format, banks can ask liquidity collectively.
    - Banks were more willing to borrow from TAF than the Discount Window.
- We analyze how *secret* bailout influences its effectiveness.
  - Gov't runs the same bailout, but Firm's acceptance is not observable.
  - If both exist, bailout recipients are pooled with the $t = 1$ holdout firms.

Secret Bailouts: No Market Revival

Suppose Gov't hides whether Firm receives bailout or not.



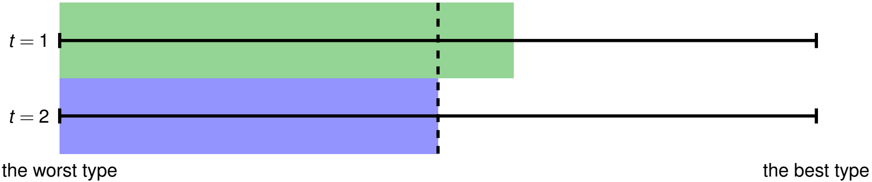
Secret bailout may not always eliminate stigma.

- Without market revival, no dynamic adverse selection arises.

Secret Bailouts: No Market Revival

Suppose Gov't hides whether Firm receives bailout or not.

No dynamic consideration for accepting bailout at $t = 1$.



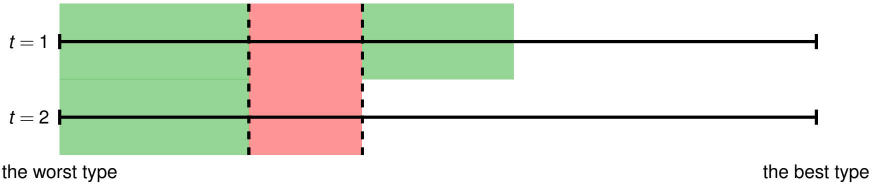
Secret bailout may not always eliminate stigma.

- Without market revival, no dynamic adverse selection arises.

Secret Bailouts: Market Revival

Suppose Gov't does not reveal identity of bailout recipients.

Holdouts are pooled together with bailout recipients.

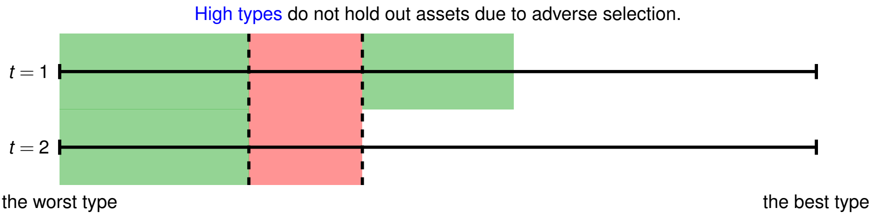


Secret bailout may not always eliminate stigma.

- Market correctly recognizes that those with high θ never attempt to sell in $t = 2$.
- This exposes only low θ as the sellers, leading to a low sale price.

Secret Bailouts: Market Revival

Suppose Gov't does not reveal identity of bailout recipients.



Secret bailout may not always eliminate stigma.

- Market correctly recognizes that those with high θ never attempt to sell in $t = 2$.
- This exposes only low θ as the sellers, leading to a low sale price.

Optimal Bailout Policy

We find which of equilibria can achieve (constrained) efficiency.

- Assume DWL $\lambda > 0$ proportional to bailout deficits:

$$S \sum_{t=1,2} \int q_t(\theta) dF(\theta) - \lambda \int_{\theta \leq \theta_g} (p_g - \theta) dF(\theta),$$

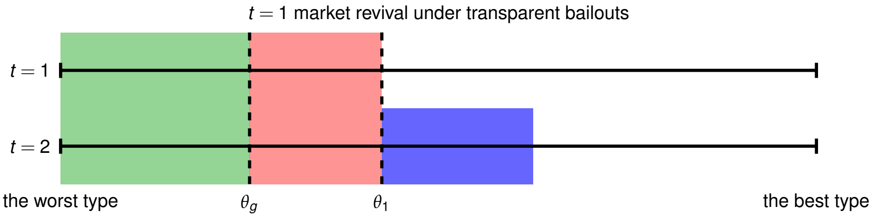
where $q_t(\theta)$ is quantity of sales in $t = 1, 2$ for each $\theta \in [0, 1]$.

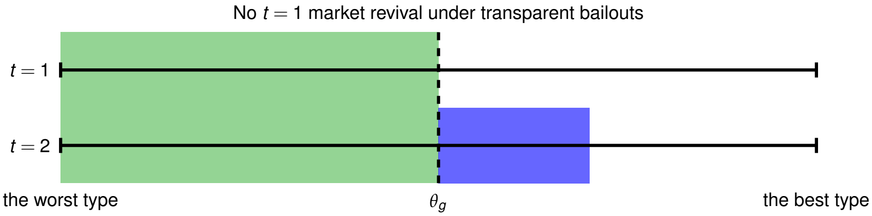
- Welfare improvements depend on compensation for recipients of bailout.

Proposition (Welfare Dominance)

1. *At the same transparency, equilibria without early market revival yield higher trades than the others.*
2. *At the same timing of market revival, secret bailout yields higher trades than the other.*

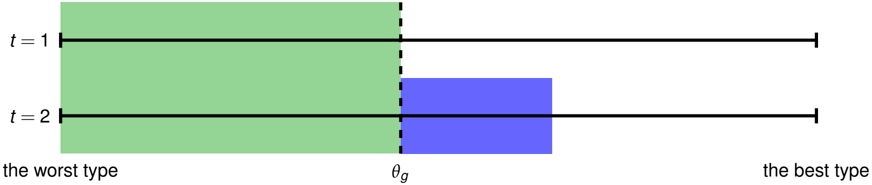
Optimal Bailout Policy



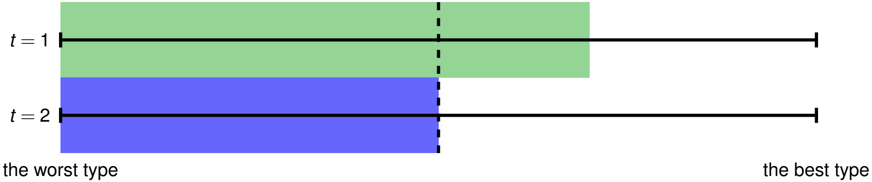


Optimal Bailout Policy

No $t = 1$ market revival under transparent bailouts



No $t = 1$ market revival under secret bailouts



Conclusion

Analysis of formation of stigma and its impacts.

- Fear of stigma discourages overall participation in bailout.
- Existence of market option has an adverse impact on overall trading.
- Effects of hiding identity vary with endogenous belief formation.

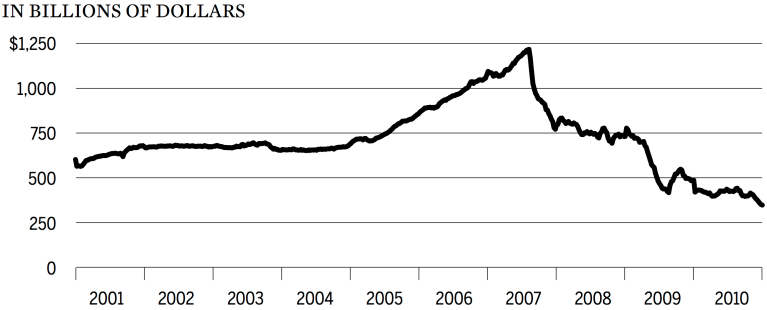
Future works:

- How to implement an optimal equilibrium outcome,
- Strategic timing of government bailouts and exit *(Chiu and Koeppl, 2011)*,

Thank You!

Asset-Backed Commercial Paper Outstanding

At the onset of the crisis in summer 2007, asset-backed commercial paper outstanding dropped as concerns about asset quality quickly spread. By the end of 2007, the amount outstanding had dropped nearly $400 billion.



NOTE: Seasonally adjusted

SOURCE: Federal Reserve Board of Governors

back to intro.

Bailout Stigma

Yeon-Koo Che   Chongwoo Choe   Keeyoung Rhee*

July 4, 2016

Abstract

We study *stigma* attached to the recipients of government bailouts via a dynamic adverse selection model, where firms' acceptance of bailouts may signal their financial weakness and worsen their subsequent funding condition. We find that: *(i)* the firms' fear of stigma leads to low take-up of bailouts and even protracted market freeze; *(ii)* severity of stigma is endogenous, resulting in multiple equilibria; *(iii)* an immediately rejuvenated market imposes negative externalities on bailout recipients by aggravating stigma; *(iv)* an equilibrium with delayed market rejuvenation welfare-dominates another equilibrium with immediate market rejuvenation; and *(v)* bailouts under controlled secrecy can implement (constrained) efficiency.

Keywords: Adverse selection, bailout stigma, secret bailout

JEL Codes: D82, G01, G18

1 Introduction

History is fraught with financial crises and large-scale government interventions, the latter often involving a highly visible, significant wealth transfer from taxpayers to banks and their creditors.

*Che: Department of Economics, Columbia University (email: yeonkooche@gmail.com); Choe: Department of Economics, Monash University (email: chongwoo.choe@monash.edu); Rhee: Department of Financial Policy, Korea Development Institute (email: ky.rhee829@gmail.com). The views expressed in this paper are solely those of the authors and should not be attributed to Korea Development Institute. Yeon-Koo Che acknowledges the financial support from the National Research Foundation through its Global Research Network Grant (NRF-2013S1A2A2035408). Heehyun Lim provided able research assistance.

According to an IMF estimate based on 124 systemic banking crises around the world during 1970 - 2007, the average fiscal costs associated with crisis management were around 13 percent of GDP (Laeven and Valencia, 2008). More recently, during the 2007 - 2009 subprime mortgage crisis, the US government paid $125 billion for assets - preferred stock and warrant - worth $86 - 109 billion to nine largest banks under the Trouble Asset Relief Program (TARP) (Veronesi and Zingales, 2010).[1] The benefits of such interventions are hard to measure since they depend on an unobservable counterfactual that would have played out in the absence of such interventions.

Philippon and Skreta (2012), and Tirole (2012) portray a plausible counterfactual in the form of market freeze and provide theoretical arguments for when and how government interventions may improve welfare. The essence of the argument is that the government can jump-start the market when severe adverse selection leads to market freeze. By cleaning up bad assets or 'dregs skimming' through public bailouts, the government can improve market confidence, thereby galvanizing transactions in healthier assets. But this argument misses an important dynamic implication of bailout. Bailout often attaches stigma to its recipients; by signalling susceptibility to shocks, they often suffer from an increased borrowing cost. Such reputational concerns in the dynamic context affect not only the costs of public bailout but also the design of optimal bailout policy.

Several observations from the TARP suggest that the fear of bailout stigma is real and significant. First, financially distressed firm often refuse bailout offers that are attractive precisely because of the fear of stigma, particularly in light of the transparency provision in the TARP that required all transactions be publicly announced within two days of execution. Although GM and Chrysler received the rescue loans under the Auto Industry Program in the TARP, Ford refused it with a view to 'legitimately portraying itself as the healthiest of Detroit's automakers' ("A risk for Ford in shunning bailout, and possibly a reward," *The New York Times*, December 19, 2008).[2][3] Second, the concern over limited participation due to stigma led the policy makers to apply unprecedented pressure on major banks that borders on coercion. At the now-famous

[1] Congressional Budget Office (2012) estimates the overall cost of the TARP at around $32 billion, the largest part of which stems from assistance to AIG and the automotive industry while capital injections to financial institutions are estimated to have yielded a net gain. For detailed assessments of the various programs in the TARP, see the *Journal of Economic Perspectives (2015)*.

[2] Such reluctance to receive government offers of recapitalization was also noted during the Japanese banking crisis of the 1990s (Corbett and Mitchell, 2000; Hoshi and Kashyap, 2010), with which the subprime mortgage crisis in the US shares a lot in common.

[3] Ford's refusal to accept a bailout was initially perceived by the market as a risky move, which was reflected in the rise in Ford's CDS spreads relative to Chrysler's. But Ford's profit and stock price showed a remarkable turnaround in 2009, part of which is attributed to the respect with customers and investors that Ford gained by refusing a bailout. (`http://www.nasdaq.com/investing/ford-turns-a-profit-after-turning-down-bailout.aspx`, accessed Nov 17, 2015).

meeting held on October 13, 2008, Henry Paulson, then Secretary of the Treasury, compelled the CEOs of nine largest banks to be the initial participants in the TARP ("Eight days: the battle to save the American financial system", *The New Yorker*, September 21, 2009). Third, participants in the TARP were eager to exit the program early, often naming stigma as the main reason. For instance, Signature Bank of New York was one of the first to repay its TARP debt of $120 million. Its chairman, Scott A. Shay, said, "We don't want to be touched by the stigma attached to firms that had taken money." ("Four small banks are the first to pay back TARP funds", *The New York Times*, April 1, 2009). It is also well known that Jamie Dimon, CEO of JP Morgan Chase, wanted to exit the TARP to avoid the stigma ("Dimon says he's eager to repay 'Scarlet Letter' TARP," *Bloomberg*, April 16, 2009).[4]

In addition, it is well-documented both empirically and anecdotally that the fear of stigma underlies banks' reluctance to use the central bank's discount window facility.[5] Peristiani (1998) provides early evidence that American banks were reluctant to borrow from the Federal Reserve's discount window even at a rate below the Federal Reserve target rate. Furfine (2001, 2003) finds similar evidence from the Federal Reserve's Special Lending Facility during 1999-2000 and the new discount window facility introduced in 2003. Armantier et al. (2011) provide more recent evidence from the 2007-2008 financial crisis utilizing the Federal Reserve's Term Auction Facility bid data and estimate the cost of stigma and its effect. Defining a bid premium over the discount window rate as the discount window stigma, they find the average stigma was 0.37 percent. Gauthier et al. (2015) go on further showing that the banks that used the Term Auction Facility in 2008 paid around 0.31 percent less in interbank lending in 2010 than those that used the discount window. Cassola, Hortaçsu and Kastl (2013) report similar findings from the bidding data from the European Central Bank's auctions of one-week loans. Krishnamurthy, Nagel and Orlov (2014) find that in the repo financing, the dealer banks with larger shares of collaterals backed by the government-sponsored enterprises – thus repayments of those collaterals are (implicitly) guaranteed – borrowed less from the Primary Dealer Credit Facility (PDCF). Their finding is consistent with the view that the attached stigma discouraged the dealer banks in good shape from using the PDCF despite its attractive funding rates. Indeed such a concern is echoed in a speech given by the former Federal Reserve chairman Ben Bernanke in 2009: 'The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness - the so-called stigma problem.'

[4]Of course, the fear of stigma is not the only reason for an early exit. Wilson and Wu (2012) find that early exit by banks is also related to CEO pay, bank size, capital, and other financial conditions.

[5]Fleming (2012) investigates how the various emergency liquidity facilities provided by the Federal Reserve during 2007-2009 were designed to overcome the limitations of traditional policy instruments at a time of the financial crisis. He also surveys the empirical literature which documents the effectiveness of those facilities.

Examples such as above raise questions about whether public bailouts are effective in the first place, and if so, how the policy should be designed in the presence of the stigma. We address these questions by extending Tirole (2012) into two periods in the most parsimonious way. There is a continuum of firms, each endowed with one unit of asset in each period. For each firm, the quality of asset in both periods is identical, which is the firm's private information. In each period, an investment opportunity with positive NPV arrives for each firm. But firms are liquidity-constrained and their projects are unpledgeable so that they need to sell their assets to finance the investment. Firms' first-period actions - whether they sell their assets, to whom, and at what terms - are observed publicly. Based on this observation, the market updates belief on the cross section of firms within each period as well as across the two periods. When the financing need arises again in the second period, the market's offer is based on its revised belief.

In the absence of government intervention, stigma is attached to those firms that choose to sell in the first period if firms with low-quality assets are more likely to sell. We call this *the early sales stigma*, which implies that those selling early expect to receive lower prices for their assets in the second period. Compared to the one-shot game, firms are more likely to delay their asset sales in the first period for fear of the early sales stigma, which renders market freeze more likely. This further justifies the case for government intervention in the first period. Government bailouts introduce another type of stigma, however. Provided that firms with low-quality assets participate in the bailout program in the first period, the market's offer in the second period would be strictly lower for those that participate in the bailout than those that held out. This difference in the market's second-period offers captures the adverse effect of stigma from accepting the bailout offer in the first period. We call this *the bailout stigma*.[6]

An obvious implication of the bailout stigma is that firms are reluctant to accept the government's offer for fear of its adverse effect in the second period. However, this does not necessarily mean that bailout is ineffective. A flip side of the stigma is the reputational gain enjoyed by those that refuse the bailout. An important way in which the bailout helps the firms is by creating this opportunity to boost reputation by refusing the bailout offer, as reflected in Ford's refusal to accept the TARP rescue loans. In fact this is a very important lesson that can be learned from our analysis of bailout in a dynamic context, which has not been well appreciated in the literature or policy analysis. But for such reputation building to be possible, there must be some firms accepting the bailout offer. To induce acceptance, bailout terms need to be attractive enough to compensate for the stigma, which makes bailout costly. We summarize below our main findings.

[6]Throughout the paper, we use the term 'stigma' to refer to both types. But the exact reference should be clear from the context.

First, if the bailout stigma is expected to be severe, firms with high-quality assets refuse to accept a bailout, which further aggravates the stigma. As a result, the bailout offer, even if strictly better than what is offered in the market, may be refused by all firms. Second, severity of the bailout stigma has self-fulfilling nature, resulting in multiple equilibrium responses to some bailout terms. Particularly, bailout terms lower than a threshold induce an equilibrium with severe bailout stigma to the extent that the subsequent market for bailout recipients freezes, as well as another equilibrium in which the market for the bailout recipients is active. The fact that bailout terms inducing the severe bailout stigma are bounded above implies that the worst equilibrium is discontinuous in bailout terms. This also suggests that there is a minimum term the government may feel compelled to offer to avoid a lackluster response. Third, when the market is immediately rejuvenated in the first period, the stigma for firms receiving the bailouts is worsened, which leads overall volume of trade to drop. More precisely, whenever there is an equilibrium with the immediate market rejuvenation, there is another welfare-superior equilibrium in which the private market is not rejuvenated in the first period. In other words, in the presence of bailout stigma, the role of public bailouts highlighted in boosting private market loses its welfare justification. Thus the dregs skimming role of bailout emphasized by Tirole takes a very different nuance, and dregs skimming may not even happen in the dynamic context.

Given that the bailout stigma stems from the transparency of the bailout program, we next ask whether secrecy may mitigate the bailout stigma and encourage participation. To study this, we consider the case where the identity of bailout recipients is not revealed to the market.[7] In this case, the market observes only those that sell assets to the market in the first period. Thus secrecy pools together the firms receiving the bailout and those that refuse to do so. On one hand, secrecy can help mitigate the stigma the former firms would suffer from under transparency, which can increase participation in the bailout. On the other hand, secrecy can dampen the reputational gain the latter firms would enjoy under transparency, eliminating the opportunity for firms to build reputation by refusing bailouts. While the precise effect of secrecy depends on different types of equilibria as in the case under transparency, our general finding is that secret bailouts lead to more participation in the first period but result in less asset trade in the second period.

Finally we discuss the welfare implications of various bailout policies and the design of an optimal bailout policy. We cast the problem in the mechanism design framework. By focusing on deterministic mechanisms in which the government intervenes only in the first period, we show that the optimal bailout mechanism has a cutoff structure: firms with low-quality assets sell in both periods; those with intermediate-quality assets sell only in the first period;

[7] How such a policy may be implemented in practice is discussed in Section 5.

and those with high-quality assets do not sell in either period. Moreover the optimal policy is implemented by a secret bailout that does not rejuvenate the market immediately. This is because secrecy eliminates the adverse effect of bailout stigma while the reputational loss the holdout firms would experience is mitigated when the market is not rejuvenated immediately. In terms of the disclosure rule, an equilibrium under secrecy dominates an equilibrium under transparency with or without immediate market rejuvenation. Under either disclosure rule, an equilibrium that does not involve immediate market rejuvenation dominates the one with immediate market rejuvenation. Thus our results suggest that market rejuvenation that would be deemed a successful outcome of government intervention in the static setting needs to be assessed more carefully in the dynamic setting with reputational concern. In the latter, policy can be more successful when it has a delayed effect on the market.

The rest of the paper is organized as follows. Section 2 contains a review of the related studies. Section 3 presents our model and offers brief discussions of equilibria without government intervention. In Section 4, we study various equilibria under government intervention. Section 5 studies the case of secret bailouts. Section 6 provides the analysis of optimal policy design while Section 7 concludes the paper. Appendix contains all the proofs not provided in the main text, and the supplementary appendix available online provides the equilibrium characterization.

2 Related Literature

While the broad theme of this paper is related to an extensive literature on the benefits and costs of government intervention in distressed banks,[8] our work is most closely related to Philippon and Skreta (2012), and Tirole (2012) which focus on the adverse selection in the asset markets as a main ground for government intervention.[9] As mentioned previously, these studies rely on static models. As a result, although relatively low types accept bailouts, the resulting stigma does not have any adverse effect in the subsequent financing. Our dynamic model not only

[8]The primary rationale for intervention is to prevent contagion of bank runs whether it stems from depositor panic (Diamond and Dybvig, 1983), contractual linkages in bank lending (Allen and Gale, 2000), or aggregate liquidity shortages (Diamond and Rajan, 2005). The costs of anticipated bailouts due to the time-inconsistency of policy are discussed, among others, in Stern and Feldman (2004).

[9]Regarding the optimal form of bailouts, Philippon and Skreta (2012) show that optimal interventions involve use of debt instruments when adverse selection is the main issue. With additional moral hazard but limits on pledgeable income, Tirole (2012) justifies asset purchases. When there is debt overhang due to lack of capital, Philippon and Schnabl (2013) find that optimal interventions take the form of capital injection in exchange for preferred stock and warrants. During the US subprime crisis, the EESA initially granted the Secretary of the Treasury authority to purchase or insure troubled assets owned by financial institutions. But the Capital Purchase Program under the TARP switched to capital injection against preferred stock and warrants.

captures bailout stigma explicitly but also shows how the role of bailout in the dynamic setting is qualitatively different from that in the static setting.

Reputational concerns by banks are explicitly considered in Ennis and Weinberg (2013), La'O (2014), and Chari, Shourideh and Zetlin-Jones (2014). In Ennis and Weinberg (2013), banks with high quality assets use interbank lending while those with low quality assets use the discount window for short-term liquidity needs. The resulting discount window stigma is reflected in the subsequent pricing of assets. In La'O (2014), financially strong banks use the Federal Reserve's Term Auction Facility since wining the auction at premium signals financial strength, which protects them from predatory trading. The main focus in Chari, Shourideh and Zetlin-Jones (2014) is to show how reputational concerns in the secondary loan markets can result in persistent adverse selection. Since all three studies consider discrete types of banks and there is no government bailout, their results are not directly comparable to ours. However, the separating equilibrium in the first two studies roughly corresponds to a special case of our equilibria where market is rejuvenated in the first period while the pooling equilibrium in the third study corresponds to our equilibrium where government crowds out the market in the first period. We provide a full characterization of all possible equilibria in our model. In addition these studies do not consider policy-related issues such as different disclosure rules.

Our paper is also related to the studies in dynamic adverse selection in general (Inderst and Müller, 2002; Janssen and Roy, 2002; Moreno and Wooders, 2010; Camargo and Lester, 2014; Fuchs and Skrzypacz, 2015) and those with a specific focus on the role of information in particular (Hörner and Vieille, 2009; Daley and Green, 2012; Fuchs, Öry and Skrzypacz, 2012; Kim, 2012).[10] The key insight in the first set of studies is that dynamic trading generates sorting opportunities, which are not available under the static market setting. But each seller has only one opportunity to trade in these studies, so signaling is never an issue in these models. The second set of studies relates to different disclosure rules and how they affect dynamic trading. For example, Hörner and Vieille (2009), and Fuchs, Öry and Skrzypacz (2012) show that secrecy (private offers) tends to alleviate adverse selection but transparency (public offers) does not. Once again, each seller has only one trading opportunity in these studies. Hence, although past rejections can boost reputation, acceptance ends the game. In contrast, each seller has three distinct signaling opportunities in our model, i.e., early sales, acceptance of bailout offer, refusal to accept the bailout offer. Although our model also shows that secret bailouts dominate transparent bailouts, the precise impact of secrecy on boosting asset trade varies with pooling structures in the first

[10]Others include dynamic extensions of Spence's signaling model with public offers (Noldeke and Van Damme, 1990), private offers (Swinkels, 1999), and private offers with additional public information such as grade (Kremer and Skrzypacz, 2007).

period. Most important, none of these papers studies government intervention into market failure.

3 Model and Preliminaries

Our model is a two-period extension of Tirole (2012). There is a continuum of firms each endowed with two units of legacy assets of the same value. The value of the asset θ is privately known to each firm which is distributed on $[0, 1]$ with distribution function F and density f. We assume that f satisfies log-concavity: $\frac{\partial^2 \log f(\theta)}{\partial \theta^2} < 0$ for all θ. Throughout, a truncated conditional expectation, $m(a, b) := \mathrm{E}[\theta | a \leq \theta \leq b]$, figures prominently in our analysis, and log-concavity of f means that $0 < \frac{\partial m(a,b)}{\partial a}, \frac{\partial m(a,b)}{\partial b} < 1$, a property we will use throughout (Bagnoli and Bergstrom, 2005). For convenience, we call a firm with legacy asset θ a type-θ firm.

In each of the two periods $t = 1, 2$, an investment project becomes available for each firm. The project requires cost I and yields strictly positive net return S, hence is socially valuable. But limited pledgeability of the project inhibits direct financing; the firm can only fund the project by selling its legacy asset to buyers in the competitive market. We assume that the firm sells at most one unit of its asset in each period,[11] and the return from the $t = 1$ project cannot be used to fund the $t = 2$ project.

The government bailout may occur prior to the firms' investment decisions. Prior to period $t = 1$, the government offers to purchase firms' assets at a fixed price p_g. Firms then decide whether to accept this offer. Having observed the government offer and firms' responses, buyers make simultaneous offers to firms, and the firms decide whether to accept one of the offers. At the end of $t = 1$, all parties observe the set of firms - but not individual types - that sold their assets in $t = 1$, whether the sale was to the government or to the market, and at what terms. The game up to this point is the same as that of Tirole (2012). Our model augments the game by introducing period $t = 2$, which repeats the $t = 1$ subgame.

Although our model is stylized, it introduces reputational considerations facing the firm in a simple way that allows for clear comparison with Tirole (2012). As will be seen, the main feature of this model is the inference that the market makes on the firm's type from its behavior in $t = 1$. Obviously, the inference is irrelevant in the one-shot model, but it now clearly affects the terms of trade in $t = 2$. Ultimately, our main focus is on how this reputational concern feeds

[11] Any equilibrium that involves firms selling more than one unit in $t = 1$ can be implemented as an equilibrium in which each firm sells one unit in each period. Such an equilibrium can be supported if the market observes the number of units each firm sells, since it can attach an unfavorable belief for those selling two units.

back into the firm's decision to accept government bailout in $t = 1$. But the reputational concern also affects the firm's decision in $t = 1$ even without the government intervention as we discuss below.

3.1 One-Period Model à la Tirole (2012)

We begin by recapitulating the key insight from Tirole (2012) by considering the case with only one period. Suppose first that there is no government intervention. In this case, the model reduces to an Akerlof's lemon market, described by Figure 1. A firm is willing to sell its asset as long as the price offer p and net investment return S (enabled by the asset sale) pays off its asset value θ. Hence, the supply curve is given by $p = \theta - S$, where $\theta - S$ is the effective reservation value of the asset to the seller. Not surprisingly, firms with low values $\theta \leq p + S$ are willing to sell. Meanwhile, the buyers must break even with respect to the "average benefit" $m(0, \theta)$ of the assets being sold, so the demand is given by $p = m(0, \theta)$.

The equilibrium is given by the intersection $\theta_0^* := \sup\{\theta' | \theta' - S \leq m(0, \theta')\}$ of the two curves at price $p_0^* = m(0, \theta_0^*)$. Log concavity of f ensures the uniqueness of the intersection point. (Note the dependence on S will be suppressed, unless needed.) Since trade is always efficient,[12] a market failure arises, and its magnitude depends on the surplus S. The market *freezes completely* ($\theta_0^* = 0$) if $S < \underline{S}_0$; the market *freezes partially* ($\theta_0^* \in (0, 1)$) if $S \in (\underline{S}_0, \overline{S}_0)$; and the market is *fully active* ($\theta_0^* = 1$) if $S > \overline{S}_0$, where $m(0, \theta_0^*(\underline{S}_0)) = I$ and $\overline{S}_0 := 1 - \mathrm{E}[\theta] > \underline{S}_0$. In the first two cases, adverse selection is severe enough to inhibit financing of socially valuable projects.

Suppose now the government offers to purchase the legacy asset at some price p_g, before the market opens. Consider $p_g \geq \max\{I, p_0^*\}$, or else the bailout will have no effect. To sharpen prediction, Tirole (2012) further makes a refinement assumption whereby the market shuts down with a vanishingly small probability.[13] Given this, there exist two cutoffs $0 \leq \theta_g \leq \hat{\theta}_0 \leq 1$ such that the types $\theta < \theta_g$ sell to the government, the types $\theta \in (\theta_g, \hat{\theta}_0)$ sell to the market, and the types $\theta > \hat{\theta}_0$ do not sell at all.

In equilibrium, with the types $[0, \theta_g]$ removed by a bailout, the remaining market reduces to the Akerlof's lemon problem on truncated types $[\theta_g, 1]$. Let $\gamma(\theta') := \sup\{\theta'' | \theta'' - S \leq m(\theta', \theta'')\}$

[12] This is seen by the fact that the marginal benefit θ (dashed line) is always above the supply curve.

[13] Formally, if the market collapses with probability $\varepsilon > 0$, a firm will sell to the market if and only if

$$(1 - \varepsilon)(p + S) + \varepsilon\theta \geq p_g + S \Leftrightarrow \theta \geq [p_g + S - (1 - \varepsilon)(p + S)]/\varepsilon,$$

where p is the price that prevails in the market.

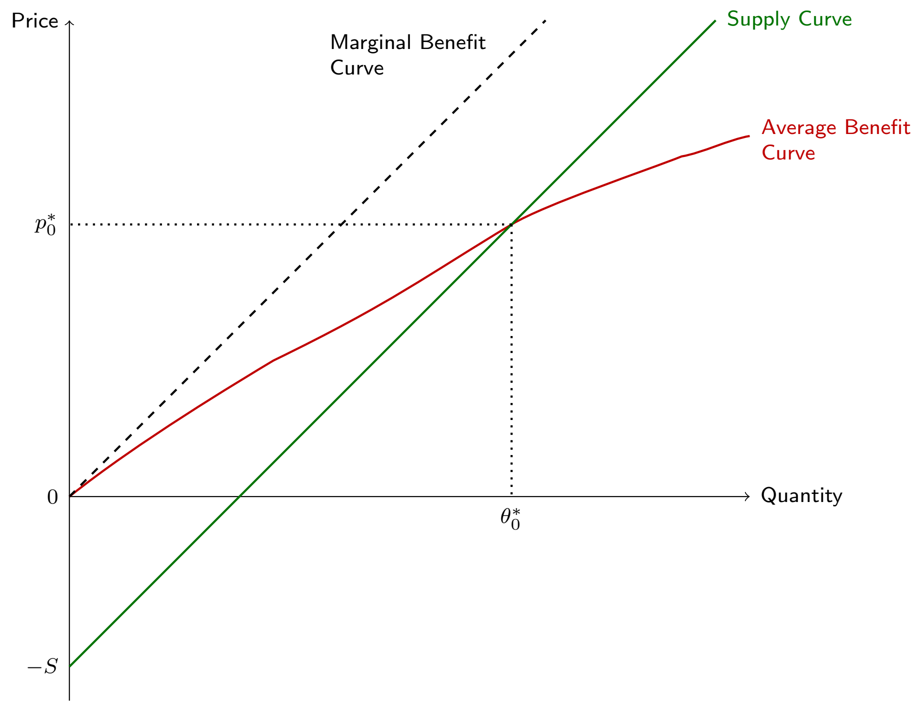


Figure 1 – One-shot Model without Bailout

denote the lemon equilibrium on truncated types $[\theta', 1]$.[14] Then, the highest type $\hat{\theta}_0$ selling to the market must equal $\gamma(\theta_g)$. Further, if both the government and market offers are accepted by positive measures of firms, we must have $p_1 = p_g$, or else the lower-price offer will not be accepted. Hence,

$$p_g = m(\theta_g, \gamma(\theta_g)). \tag{1}$$

Given the log-concavity of f, the critical cutoff type θ_g satisfying (1) is well defined for $p_g \in [p_0^*, 1]$.[15] Most important, whenever $p_g > \max\{I, p_0^*\}$, we have $\theta_g > 0$, hence $\gamma(\theta_g) > \gamma(0) = \theta_0^*$. In other words, bailout improves the asset trading and therefore the financing of socially valuable projects whenever the market is not fully active absent bailout—hence the welfare rationale for the bailout.

Proposition 1. *(Tirole, 2012) If the government offers to purchase the legacy asset at price $p_g \geq \max\{I, p_0^*\}$, then types $\theta < \theta_g$ sell to the government and types $\theta \in (\theta_g, \gamma(\theta_g))$ sell to the market at the same price, where θ_g is given by (1). Any offer $p_g > \max\{I, p_0^*\}$ increases the*

[14] In terms of Figure 1, the truncation shifts up the average benefit curve: its starting point moves along the marginal benefit curve by θ'. Consequently, the intersection point shifts out; i.e., $\gamma(\theta')$ increases in θ'.

[15] It is easy to see $\gamma(\cdot)$ is continuous and nondecreasing. Since the m is continuous and increasing in both arguments (due to log concavity of f), $m(0, \theta_0^*) = p_0^* \leq p_g \leq 1 = m(1, 1)$, θ_g is well defined for $p_g \in [p_0^*, 1]$.

volume of trade and financing whenever $S < \overline{S}_0$.

According to Proposition 1, the government is picking up the most "toxic" assets; this improves the market's perception of the remaining assets and increases the trading of assets beyond what is possible without the bailout. Through such "dregs skimming", the government runs deficit (per unit asset) equal to $p_g - m(0, \theta_g)$, and induces trade in additional assets $\theta \in (\theta_g, \gamma(\theta_g))$. Since these additional assets are traded at the break-even price $m(\theta_g, \gamma(\theta_g))$ whether they are sold to the market or to the the government, the government runs the same amount of deficit whether or not the private market is involved.[16] Thus dregs skimming is desirable not because it reduces the government deficit by inducing the market's participation, but because it galvanizes trading in the remaining assets whether they are sold to the government or to the private market; without dregs skimming, the market's offer will be lower, hence less amount of assets will be sold.

3.2 Two-Period Model without Government Intervention

Unlike the one-shot problem, our two-period model introduces a signaling motive on the part of firms, since their trading behavior in $t = 1$ affects the market's belief on their assets and thus the offers they receive in $t = 2$. As is well known, signaling games admit multiplicity of equilibria. Accordingly, we focus on perfect Bayesian equilibria (in pure strategies) but impose several additional properties. First, we assume that firms discount their $t = 2$ payoff arbitrarily slightly.[17] Much in the same spirit of Tirole (2012) this assumption produces a natural sorting of firm types in terms of the timing of trading, with low types selling before high types. Second, as is standard with signaling games, we invoke D1 refinement. D1 requires that, upon an off-the-path signal being sent, uninformed players (buyers in our model) attribute the deviation to types that have most to gain from that deviation (in terms of the set of responses following that deviation).[18] In addition to ruling out implausible equilibria, this refinement ensures the equilibrium to vary

[16] Even if the government were to shut down the private market entirely (say by banning the market), this would not change the set of firms selling their assets. Those that would have sold to the market would now sell to the government: since a firm will sell to the government if and only if $\theta \leq p_g + S$, and since $p_g = p_1$ in equilibrium with the active private market, the types selling their assets are precisely the same.

[17] Formally, we focus on a limit of the perfect Bayesian equilibria of a sequence of games in which players discount $t = 2$ by $\delta \in (0, 1)$, as $\delta \to 1$.

[18] The D1 refinement can be described formally as follows. Let $U^*(\theta)$ be the payoff for type θ in a "putative" equilibrium, and let $u(r, s; \theta)$ be the payoff for type θ when it sends an "off-the-path" signal s and elicits response r as a consequence. Let $D(\theta|s) := \{r | u(r, s; \theta) \geq U^*(\theta)\}$ be the set of possible responses that would yield payoff for type θ that dominates its equilibrium payoff. Upon an off-the-path signal s being sent, the D1 refinement requires that the belief of the uninformed players be supported on the types θ for whom $D(\theta|s)$ is maximal, i.e., on types $\Theta(s) := \{\theta \in \Theta | \nexists \theta' \text{ s.t. } D(\theta'|s) \supsetneq D(\theta|s)\}$. See Fudenberg and Tirole (1991).

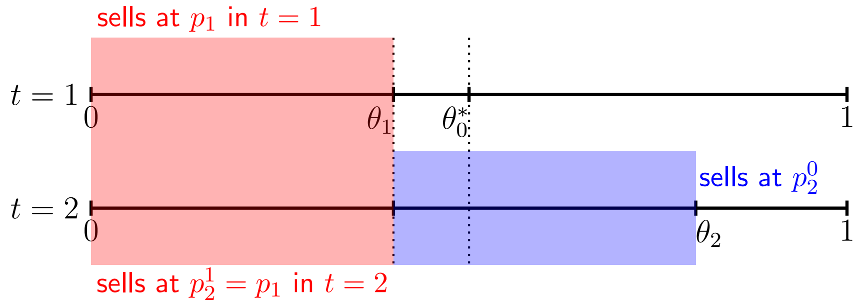


Figure 2 – Equilibrium without Government Intervention

continuously with parameter values. Third, we focus on an equilibrium in which buyers earn zero payoff. Unlike the one-shot model, zero profit is not a necessary implication of equilibrium even with D1 refinement in our model, but any equilibrium violating this property rests on an unreasonable off-path belief.[19] We call a perfect Bayesian equilibrium with these properties simply an "equilibrium." We now show that any (such) equilibrium has a cutoff structure:

Lemma 1. *In any equilibrium, there is a cutoff* $0 \leq \theta_1 \leq 1$ *such that all types* $\theta \leq \theta_1$ *sell in each of the two periods at price* $m(0, \theta_1)$ *and all types* $\theta > \theta_1$ *hold out in* $t = 1$ *and are offered price* $m(\theta_1, \gamma(\theta_1))$ *in* $t = 2$*, which types* $\theta \in [\theta_1, \gamma(\theta_1))$ *accept. If* $\theta_1 = 1$*, then* $S \geq 2(1 - \mathrm{E}[\theta])$*.*

Proof. See Appendix A. *Q.E.D.*

A typical equilibrium is shown in Figure 2 where the $t = 1$ price is denoted by p_1, the $t = 2$ price for those that sold in $t = 1$ by p_2^1, and the $t = 2$ price for those that did not sell in $t = 1$ by p_2^0. (The subscript refers to the period, and the superscript refers to whether trade occurred in $t = 1$, with 0 encoding "no trade" and 1 encoding "trade.") Intuitively, those selling in $t = 1$ have low values $\theta < \theta_1$ and those holding out have higher values $\theta > \theta_1$. Since firms' actions in $t = 1$ are observed by the market in $t = 2$, they are treated differently, with the holdouts receiving higher offer than the early sellers. Specifically, the early sellers receive price $p_2^1 = m(0, \theta_1) = p_1$ (lemons equilibrium on types $[0, \theta_1]$), and the holdouts receive price $p_2^0 = m(\theta_1, \gamma(\theta_1))$ (lemons equilibrium on truncated types $[\theta_1, 1]$). The price difference $p_2^0 - p_2^1 = m(\theta_1, \gamma(\theta_1)) - m(0, \theta_1)$ constitutes the "early sales stigma."

[19] In any such an equilibrium, buyers offer a low price in $t = 1$ that leaves them with strictly positive profit. And no buyer deviates to a higher price offer for fear that such an offer will be rejected since firms in turn believe that accepting it signals the worst type $\theta = 0$ to the market in $t = 2$, unless the deviation offer is exceptionally generous to be acceptable for some types despite such an unfavorable belief, in which case the deviation would result in a loss to the deviating buyer. Such a belief, while consistent with D1, is implausible since any price increase must be acceptable for firms with weakly higher types.

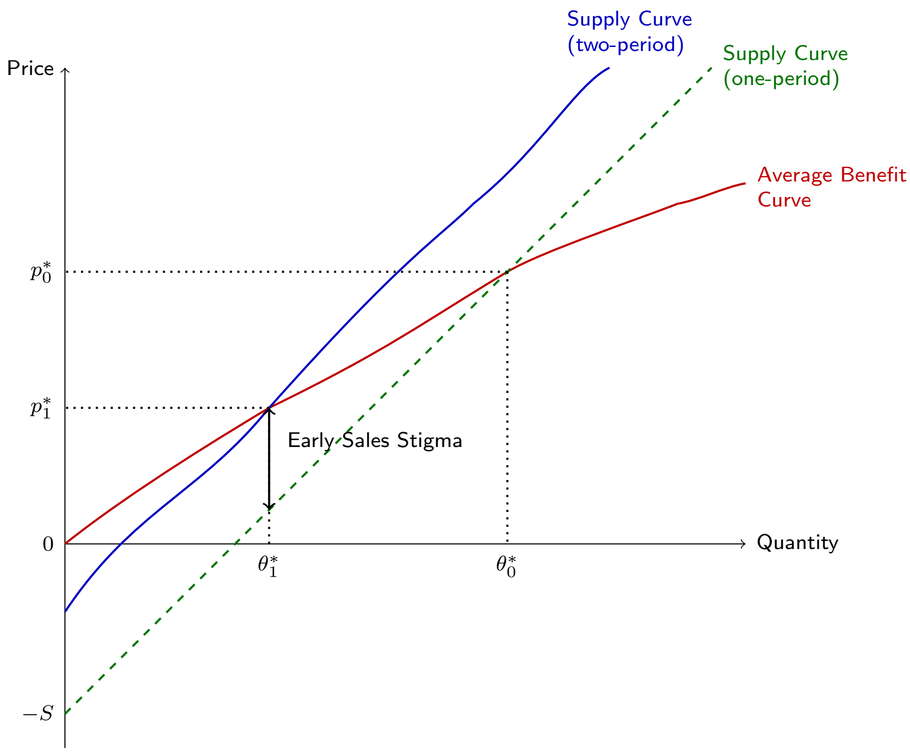


Figure 3 – $t = 1$ Market under Early Sales Stigma

How the early sales stigma affects firms' $t = 1$ incentives can be seen in Figure 3. Just like Figure 1, the buyers' demand is given by the average benefit $m(0, \theta)$ of the assets being sold, where θ is the highest-type asset on the market. The early sales stigma adds to firms' reservation value, so the supply curve shifts up by the amount equal to the stigma, and is given by $p = \theta - S + [m(\theta_1, \gamma(\theta_1)) - m(0, \theta_1)]$. Adverse selection is worsened by signaling at the equilibrium—the intersection of the supply and average benefit curves: the equilibrium trade shrinks relative to the one-shot model.

The equilibrium analysis is facilitated by the following assumption, which essentially guarantees that the supply curve crosses the average benefit curve at most once:[20]

Assumption 1. *(i)* $\Delta(\theta; S) := m(0, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(0, \theta))$ *is strictly decreasing in* θ*; (ii) If* $\Delta(0; S) \geq 0$*, then* $\Delta(0; S') > 0$ *for* $S' > S$*.*

We now provide the main result of this section.

[20] A variety of standard distributions used in economic analysis satisfy these assumptions, such as truncated normal distributions on the interval $[0, 1]$, beta distributions with various values of the shape parameters, and the uniform distribution on $[0, 1]$. Equilibrium characterization without these assumptions is more cumbersome and adds no significant insight.

Proposition 2. *(i) There is an equilibrium in which firms with* $\theta \leq \theta_1^*$ *sell at price* $p_1^* := m(0, \theta_1^*)$ *in both periods, firms with* $\theta \in (\theta_1^*, \theta_2^*)$ *sell only in* $t = 2$ *at price* $p_2^* := m(\theta_1^*, \theta_2^*)$, *and firms with* $\theta > \theta_2^*$ *never sell, where* θ_1^* *and* θ_2^* *are defined by* $\Delta(\theta_1^*; S) = 0$ *and* $\theta_2^* = \gamma(\theta_1^*)$, *respectively. We have* $\theta_1^* \leq \theta_0^* \leq \theta_2^*$, *and thus* $p_1^* \leq p_0^* \leq p_2^*$, *where the inequalities hold strictly if the cutoff in the one-period model satisfies* $\theta_0^* \in (0, 1)$. *Given Assumption 1-(i), there is at most one such equilibrium with an interior* θ_1^*.

(ii) Assume Assumption 1-(ii). In the equilibrium of (i), the $t = 1$ *market is fully active if* $S \geq \overline{S}^*$ *and suffers from partial freeze if* $S \in (\underline{S}^*, \overline{S}^*)$ *and full freeze if* $S < \underline{S}^*$, *where* $\underline{S}^*$ *and* $\overline{S}^*$ *are defined by* $\Delta(0; \underline{S}^*) = 0$ *and* $\Delta(1; \overline{S}^*) = 0$, *respectively, and satisfy* $\underline{S}^* > \underline{S}_0$ *and* $\overline{S}^* > \max\{\overline{S}_0, \underline{S}^*\}$.

(iii) In addition, there is an equilibrium with full market freeze in $t = 1$ *for any* S.

Proof. See Appendix A. *Q.E.D.*

The above proposition shows that the equilibrium trade is smaller in $t = 1$ but larger in $t = 2$ than in the one-period model. The reduced trade in $t = 1$ explains the increased range of S's for which the $t = 1$ market freezes. Meanwhile, the flip side of the reputational loss from early sales is the reputational gain the firm would enjoy by "refusing to sell" in $t = 1$. This reputational gain leads to a better term and thus mitigates adverse selection in $t = 2$, resulting in larger trade than in the static model. The dynamic trading pattern is reminiscent of those found in dynamic adverse selection models (Fuchs, Öry and Skrzypacz, 2012; Fuchs and Skrzypacz, 2015); but, the signaling motive is absent in these models since informed players trade only once.

Strikingly, our model also admits an equilibrium in which the market completely shuts down in $t = 1$, regardless of the intrinsic severity of adverse selection (i.e., the value of S). This phenomenon, which has no analogue in Tirole (2012) or in the dynamic adverse selection models, results from an interaction between signaling and adverse selection. In this equilibrium, the buyers refrain from making *any* viable offer in $t = 1$, for fear that firms accepting that offer may suffer from extreme stigma (signaling of $\theta = 0$ say) so that either the offer is rejected altogether, or if it is accepted *despite the extreme stigma* it could only mean that the quality of assets sold must be too low to be profitable for the buyers (adverse selection). Our dynamic model thus identifies a novel form of market failure that results from extreme stigma feeding extreme adverse selection.[21]

[21] Importantly, the extreme stigma is consistent with D1 refinement, although its application in the current context is somewhat unusual; note that any signaling arises only off the path when a buyer deviates and makes an offer.

4 Dynamic Adverse Selection and Bailout Stigma

In this section, we study government bailout of the firms via purchase of their assets. Specifically, we augment the game in the previous section by adding period $t = 0$ in which the government announces an offer to purchase one unit of the asset at price $p_g > 0$, and firms decide whether or not to accept the offer.[22] The game described in the previous section is then played. In each period, firms selling assets at price weakly greater than I finance their projects for that period and collect net surplus S. Whether a firm sold its asset to the government or to the market and at what terms are publicly observed at the end of $t = 1$.

For any bailout offer p_g, we consider perfect Bayesian equilibria of the ensuing subgame satisfying the afore-mentioned refinements, plus Tirole (2012)'s refinement—namely, that the private market shuts down with an arbitrarily small probability. Again, we call the resulting concept simply an "equilibrium," whose structure is characterized as follows:

Lemma 2. *In any equilibrium, there are four possible cutoffs $0 \le \underline{\theta} \le \tilde{\theta} \le \hat{\theta} \le \check{\theta} \le 1$ such that types $\theta < \underline{\theta}$ sell in both periods with first-period sales to the government, types $\theta \in (\underline{\theta}, \tilde{\theta})$ sell in both periods to the market, types $\theta \in (\tilde{\theta}, \hat{\theta})$ sell only in $t = 1$ to the government,types $\theta \in (\hat{\theta}, \check{\theta})$ sell only in $t = 2$ to the market, and types $\theta > \check{\theta}$ sell in neither periods.*

Proof. See Appendix A. *Q.E.D.*

This lemma rests on several observations. First, the firms' preferences satisfy the single-crossing property, implying that a lower type has more incentive to sell its asset; this implies that the total volume of trade must be nonincreasing in θ in any equilibrium. Second, the fact that buyers (either the government or the market) never ration their purchase means that the trade decision must be either zero or one in each period. Third, an (arbitrarily small) discounting of the second period payoff, along with the first two observations, imply that, among those who trade only for one period, the early traders have lower types than late traders. Finally, a small probability of market collapse, as in Tirole (2012), implies that the early traders choose the government ahead of the market insofar as they sell assets in both periods.

Based on Lemma 2, we provide below a complete characterization of all possible equilibria, which we group into three types. In the first type, the government's bailout offer is not accepted

[22] We assume that the government offers bailout only in the first period and for one unit of asset. This is consistent with the observed practice: governments refrain from engaging in long-term bailouts and from complete "nationalization" of distressed firms (which would be equivalent to purchasing two units of asset in our model). Further, our goal is to study the reputational consequence of taking the government bailout, which can be studied most effectively when no government bailout is available in the second period.

by any firms in the sense that $\underline{\theta} = 0$ and $\tilde{\theta} = \hat{\theta}$. In the second type, the government's offer is taken up by some firms but it does not rejuvenate the market in $t = 1$ in the sense that $\underline{\theta} = \tilde{\theta}$. In the third type, the government's offer is accepted by some firms, which also rejuvenates the market in $t = 1$, i.e., $\underline{\theta} < \tilde{\theta}$.

4.1 Equilibria in the Presence of Bailout

4.1.1 No Response Equilibrium (NR)

First, a bailout may attract "no takers" in equilibrium. "No response" would not be a surprising outcome if the offer is sufficiently unattractive, for instance, if p_g is less than p_1^*, the $t = 1$ market price without government intervention. More interestingly, "no response" may arise even when the government offers a strictly better term than the market.

The reason for this is bailout stigma. To see this, recall the equilibrium without government intervention described in Proposition 2, and suppose that equilibrium involved an active $t = 1$ market with price p_1^* and $t = 2$ market with price p_2^* for those that refused to sell in $t = 1$, where $p_2^* = m(\theta_1^*, \theta_2^*) > m(0, \theta_1^*) = p_1^*$.

Suppose now the government offers $p_g \in (p_1^*, p_2^*]$. It is an equilibrium for all firms to reject that offer, given the (out-of-equilibrium) belief by the $t = 2$ market that any takers of the bailout must have $\theta = 0$. To see this, consider any type θ. If it rejects the government offer (as suggested by the candidate equilibrium strategy), then its payoff is no less than

$$\theta + p_2^* + S \tag{2}$$

since the type has an option not to sell its asset in $t = 1$ and sell it at p_2^* in $t = 2$. If the same firm accepts the bailout, then given the out-of-equilibrium belief, its payoff is at most

$$\theta + p_g + S, \tag{3}$$

since it sells at p_g and finance its project in $t = 1$, but can at best consume its asset and realize θ in $t = 2$. Comparing (3) with (2) shows that, so long as $p_g \le p_2^*$, the firms will never accept the bailout, given the extreme stigma. Importantly, the extreme stigma is not unreasonable in the sense that it does satisfy D1. Intuitively, whenever any type gains from accepting a bailout, the worst type must also gain from accepting it, which makes it compelling for the market to attach the extreme stigma to accepting firms.

Proposition 3. *If the government offer is $p_g \leq p_2^*$, then in equilibrium no firms accept the government offer. That is, the equilibria in Proposition 2 remain equilibria for any bailout offer $p_g \leq p_2^*$.*

4.1.2 Bailout Equilibria without Immediate Market Rejuvenation

Of particular interest is an equilibrium in which the government bailout offer is accepted by some types of the firm. Such an equilibrium may exist for $p_g \geq I$. But the equilibrium may involve no active market in $t = 1$; that is, no firms sell in the private market in $t = 1$. We explore this type of equilibrium here. By Lemma 2, the equilibrium must then have a cutoff $\theta_g \in (0, 1]$ such that the firms with $\theta \leq \theta_g$ accept the bailout, and the firms with $\theta > \theta_g$ never sell their assets in $t = 1$. That the market is not active in $t = 1$ in such an equilibrium is in sharp contrast to Tirole's one-shot model, where any equilibrium with active bailout *must* also involve an active private market—namely, some positive measure of firms must also sell their assets to the market.

Bailout equilibria without an active market in $t = 1$ can be supported by an extreme early sales stigma – namely, an out-of-equilibrium belief that any buyer accepting the market's offer in $t = 1$ must have $\theta = 0$. Indeed such an out-of-equilibrium belief can be shown to satisfy D1 refinement for the equilibrium dubbed below the bailout equilibrium with severe stigma. However for another type dubbed the bailout equilibrium with moderate stigma, the out-of-equilibrium belief satisfying D1 is less extreme in that any buyer accepting the market's offer in $t = 1$ is the highest type that accepts the bailout and also sells its asset in $t = 2$. Importantly, these equilibria may arise even when the private market would develop in $t = 1$ without government intervention (i.e., with $S > \underline{S}^*$). In that case, the government bailout "crowds out" the market in $t = 1$. We show below these two types of equilibria correspond to the different degrees of endogenous stigma associated with the bailout.

□ **Bailout Equilibrium with Severe Stigma (BSS):** This is an equilibrium in which the stigma attached to the firms receiving the bailout in $t = 1$ is so severe that they never see a viable market materializing for their remaining asset in $t = 2$.

To consider such an equilibrium, suppose again types $\theta < \theta_g$ accept the bailout, and types $\theta > \theta_g$ refuse the bailout, for some threshold θ_g. In $t = 2$, the latter types will be offered a high price $m(\theta_g, \gamma(\theta_g))$, but the former types attract at most the price of $\min\{m(0, \theta_0^*), m(0, \theta_g)\} =$

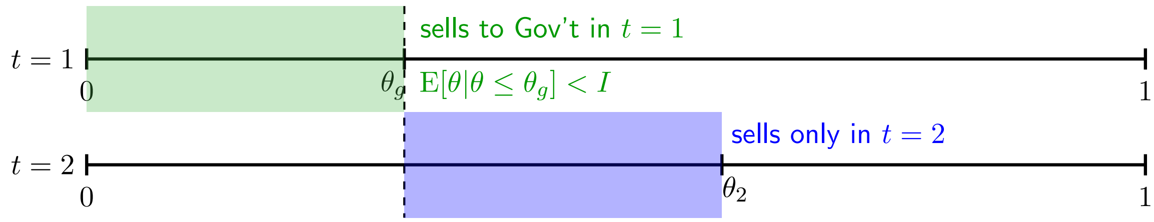


Figure 4 – Bailout Equilibrium with Severe Stigma (BSS)

$m(0, \theta_g \wedge \theta_0^*)$.[23] Hence, if

$$m(0, \theta_g \wedge \theta_0^*) < I, \tag{4}$$

then the bailout recipients can never fund their investment project, so they will not sell their assets in $t = 2$. The equilibrium of this form, labelled BSS, is depicted in Figure 4.

In equilibrium, the threshold type θ_g must be indifferent in its choice.[24] If a type-θ_g firm refuses a bailout, its payoff is

$$\Pi_{\text{refuse}}^{BSS}(\theta_g) := \theta_g + m(\theta_g, \gamma(\theta_g)) + S, \tag{5}$$

since it realizes θ_g in $t = 1$ but enjoys a higher market price $m(\theta_g, \gamma(\theta_g))$. If it accepts the bailout, then its payoff is

$$\Pi_{\text{accept}}^{BSS}(\theta_g) := p_g + S + \theta_g, \tag{6}$$

since its $t = 2$ market collapses given (4), so its only option is to consume θ_g. The indifference of the threshold type requires (6) to equal (5), or $\theta_g = \theta_g^{BSS}$, where

$$m(\theta_g^{BSS}, \gamma(\theta_g^{BSS})) = p_g. \tag{7}$$

If θ_g^{BSS} satisfies (4), then a BSS equilibrium exists. As was seen in (1), the threshold θ_g^{BSS} is well-defined, and it is nonnegative for any $p_g \geq p_0^* = m(0, \theta_0^*) = m(0, \gamma(0))$. Meanwhile, (4) will hold if θ_g^{BSS} or θ_0^* is sufficiently small. The latter is the case if $S \leq \underline{S}_0$, in which case a BSS equilibrium exists regardless of p_g. If p_g exceeds p_0^* but is close to p_0^*, then θ_g^{BSS} is sufficiently

[23] If $\theta_g > \theta_0^*$, then the standard lemons problem applies to types $\theta \in [0, \theta_g]$ that sold to the government in $t = 1$ and only the types $\theta \in [0, \theta_0^*]$ will be able to sell in $t = 2$. Thus the price offered to the $t = 1$ bailout recipients is $m(0, \theta_g \wedge \theta_0^*)$.

[24] In principle, the type could be $\theta_g = 1$ and may strictly prefer bailout. For expositional simplicity, we assume that p_g is such that the firm is never induced to strictly prefer bailout; this latter possibility would also be suboptimal for a government that incurs shadow cost of spending.

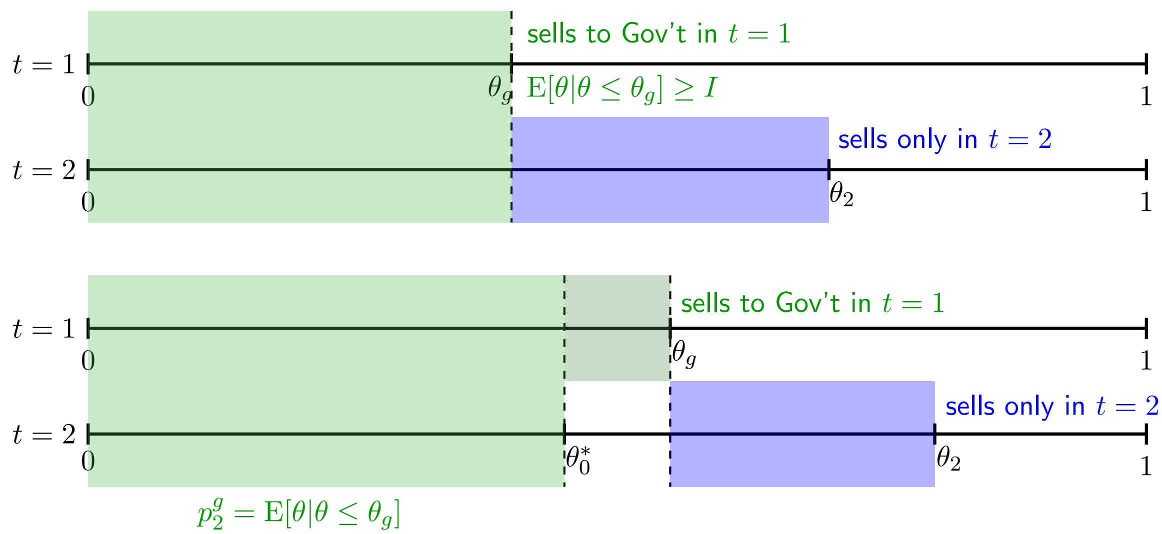


Figure 5 – Bailout Equilibria with Moderate Stigma (BMS)

close to zero, so (4) will hold. Hence, there exists $\overline{p}_g^{BSS} > p_0^*$ such that a BSS equilibrium exists for $p_g \in [p_0^*, \overline{p}_g^{BSS}]$.[25] Recall $p_1^* \leq p_0^*$ from Proposition 2-(i). This means that a BSS equilibrium exists for a range of bailout offer p_g that exceeds the market offer that would prevail without the bailout.

□ **Bailout Equilibrium with Moderate Stigma (BMS):** This is an equilibrium in which the stigma attached to the firms receiving the bailout in $t = 1$ is not so severe that a viable market for their remaining assets materializes in $t = 2$. The equilibrium of this type, labeled BMS, is depicted in Figure 5.

To consider a BMS equilibrium, suppose again that firms with $\theta \leq \theta_g$ accept the bailout offer p_g but those with $\theta > \theta_g$ refuse it and never sell their assets in $t = 1$. Importantly, suppose now

$$m(0, \theta_g \wedge \theta_0^*) \geq I, \tag{8}$$

so the break-even price offered in $t = 2$ to the bailout recipients is high enough to fund their $t = 2$ projects.

Given this, the payoff to type-θ_g firm from rejecting bailout in $t = 1$ is the same as (5), but its payoff from accepting the bailout is now given by

$$\Pi_{\text{accept}}^{BMS}(\theta_g) := p_g + S + \max\{\theta_g, m(0, \theta_g \wedge \theta_0^*) + S\}, \tag{9}$$

[25] The upper bound $\overline{p}_g^{BSS}$ is defined to be p_g that makes θ_g^{BSS} satisfy (4) with equality.

since the firm now can sell the remaining asset in $t = 2$ at price $m(0, \theta_g \wedge \theta_0^*)$. Type θ_g-firm is indifferent, or $\theta_g = \theta_g^{BMS}$, a value at which (9) equals (5). Given the assumption of strict log concavity of density $f(\cdot)$ made in Section 3, θ_g^{BMS} is well-defined.[26] If θ_g^{BMS} satisfies (8), then a BMS equilibrium exists. As was seen before, if $S \leq \underline{S}_0$, then (8) can never hold, so a BMS equilibrium does not exist. But if $S > \underline{S}_0$, then a BMS equilibrium exists for $p_g \geq \underline{p}_g^{BMS}$, where $\underline{p}_g^{BMS}$ is the bailout offer that induces (8) to hold with equality at $\theta_g = \theta_g^{BMS}$.

We now provide more precise characterization of these two equilibria and compare them.

Proposition 4. *(i) If $S > \underline{S}_0$, then $\underline{p}_g^{BMS} < \overline{p}_g^{BSS}$, so for each bailout offer $p_g \in [\underline{p}_g^{BMS} \vee p_0^*, \overline{p}_g^{BSS})$ both the BSS and BMS equilibria exist.*

(ii) The BMS equilibrium induces more asset trade than the BSS equilibrium and the equilibrium without government bailout (when the relevant equilibria exist). The BSS equilibrium induces more asset trade than the full freeze equilibrium without bailout, but for $S > \underline{S}^*$, the BSS equilibrium under some bailout offer p_g may lead to less trade than without bailout.

Proof. See Appendix A. *Q.E.D.*

A few implications are worth exploring. First, the two equilibria exhibit a difference in the sustainability of the bailout benefit. In the BSS equilibrium, the effect of bailout is "short-lived," in the sense that the bailout recipients are unable to secure financing on their own in $t = 2$. By contrast, in the BMS equilibrium, the bailout does not dampen the reputation of the recipients too much so that they can obtain financing for their $t = 2$ projects. The sustainability of bailout effect is of an important policy concern, yet it has not been addressed in the existing research due to their static framework. Our analysis identifies the degree of bailout stigma as an important determinant of the long-term sustainability of the bailout effect.

Second, the degree of bailout stigma depends bot only on p_g (the attractiveness of bailout) but also on the endogenous belief of the market. In particular, the possible coexistence of BMS and BSS equilibria stated in (i) is due to the endogeneity of the belief: If a severe bailout stigma exists, its recipients cannot fund their $t = 2$ projects; this makes the bailout unattractive and induces only low types of firms to accept the bailout; and this in turn validates the severe stigma, leading to a BSS equilibrium. If on the other hand firms expect more moderate stigma, they can expect to be able to fund their $t = 2$ projects, which encourages high type firms to participate in the bailout, thus validating the moderate stigma as well as the BMS equilibrium. Formally,

[26] If $\theta_g \leq \theta_0^*$, θ_g^{BMS} is a solution to $p_g + S = \theta_g - m(0, \theta_g) + m(\theta_g, \gamma(\theta_g))$. If $\theta_g > \theta_0^*$, θ_g^{BMS} is a solution to $p_g = m(\theta_g, \gamma(\theta_g))$. The property of strict log concavity of $f(\cdot)$ implies: *(i)* $\theta - m(0, \theta)$ is continuous and increasing in θ; *(ii)* $\gamma(\theta)$ is continuous and increasing in θ. These features guarantee existence and uniqueness of θ_g^{BMS}.

the multiplicity can be seen by the fact that for each $\theta_g < \theta_0^*$, the payoff (9) from accepting the bailout is higher under the BMS equilibrium than the corresponding payoff (6) under the BSS equilibrium, which suggests that $\theta_g^{BMS} < \theta_g^{BSS}$. It is thus possible (and formally stated above) that some bailout offer p_g may admit θ_g^{BMS} and θ_g^{BSS} that respectively satisfy (8) and (4), supporting both types of equilibria. It also explains why the BMS equilibrium supports more trade than the BSS equilibrium, when they both exist.

Finally, as pointed out in (ii), bailouts may promote trading and the funding of projects. The channel of the effect can be subtle and indirect. In the two equilibria, the firms that refuse bailout in $t = 1$ experience improved market perception about their assets and are thus more able to finance their projects in $t = 2$. In other words, the beneficial effect of a bailout may accrue to firms that do not directly participate in the bailout. This delayed and indirect nature of the benefit would be difficult to appreciate for an outside observer, but may be nevertheless important.

4.1.3 Bailout Equilibrium with Market Rejuvenation (MR)

An important role of government bailout highlighted by Tirole (2012) was to "prop up" the private market. As was seen in Section 3.1, a government bailout takes the most toxic assets out of the system, and thus improves the market perception of the remaining assets and brings confidence to private investors seeking to buy them.

We argue here this "dregs skimming" role of government bailout can be fundamentally undermined by bailout stigma. First, the bailout need not take out the worst types of the asset, which limits its ability to boost market confidence. More important, the immediate revival of market worsens bailout stigma, which in turn raises the cost of the bailout for the government, forcing it to pay a (possibly significant) premium over a market price for the asset. In fact, the stigmatizing effect of the market revival is so strong as to render market revival completely undesirable. We will show below that, for any equilibrium in which the $t = 1$ market is active, there is an equilibrium without market revival that supports a higher total trade and investment. In short, "turning off" the private market makes bailout more effective.

To begin, again fix a bailout offer p_g, and consider a possible equilibrium with market revival in $t = 1$. For a relatively low range of p_g, such an equilibrium is characterized by three cutoffs, $0 < \theta_g < \theta_1 < \theta_2 \leq 1$ such that in $t = 1$ the firms with $\theta < \theta_g$ accept the bailout, those with $\theta \in (\theta_g, \theta_1]$ sell to the market at the break-even price $m(\theta_g, \theta_1)$, and those with $\theta > \theta_1$

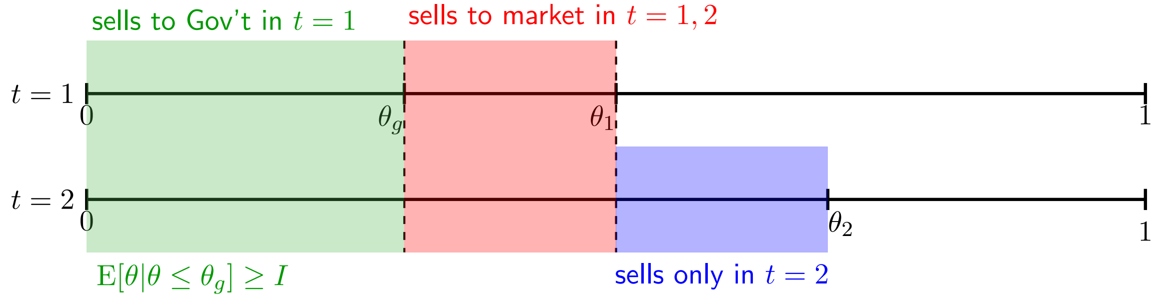


Figure 6 – Bailout Equilibrium with Market Rejuvenation (MR1)

hold out. In $t = 2$, the bailout recipients sell to the market at the break-even price $m(0, \theta_g)$,[27] the sellers to the market sell again at the same price, and the $t = 1$ holdouts are offered a higher price $m(\theta_1, \gamma(\theta_1))$ where $\theta_2 = \gamma(\theta_1)$. The critical types θ_g, θ_1 and θ_2 must satisfy relevant indifference conditions, and the bailout offer p_g must be in the right range to support this type of equilibrium.[28] This type of equilibrium, labeled **MR1**, is depicted in Figure 6.

The MR1 equilibrium at first glance exhibits the dregs skimming role of bailouts highlighted by Tirole (2012): The government bailout is accepted by the worst types of the firm, leaving the remaining types to be served by the market at a term better than what they would get otherwise. A closer inspection reveals the cost of reviving the market in $t = 1$, however. The fact that the government bailout attracts the worse types than the market means that the bailout recipients are subject to a severe stigma in the $t = 2$ market. For the bailout to be acceptable, the bailout offer p_g must compensate its recipients for the loss from stigma. Thus, unlike Tirole (2012), the bailout term includes the premium over market that makes up for the stigma $m(\theta_g, \theta_1) - m(0, \theta_g)$. This means that the cost of inducing trade is much higher than in

[27] The possibility of the bailout recipients not being able to sell in $t = 2$ – i.e., $m(0, \theta_g) < I$ – is incompatible with market revival in $t = 1$. In equilibrium, those types $\theta \in (\theta_g, \theta_1]$ sell their asset to the market in both $t = 1$ and $t = 2$. By Lemma 2, then the recipients of bailout, which have types $\theta \in (0, \theta_g]$, must also sell their remaining asset in $t = 2$.

[28] First, type-θ_g firm must be indifferent between accepting bailout and selling to the market:

$$p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S, \tag{10}$$

where the LHS is type-θ_g firm's payoff from accepting the bailout, and the RHS is its payoff from selling its asset to the market at the break-even price. Second, type-θ_1 firm is indifferent between selling to the market and holding out:

$$2m(\theta_g, \theta_1) + 2S = \theta_1 + m(\theta_1, \gamma(\theta_1)) + S, \tag{11}$$

where the second term of the RHS corresponds to the higher market price that would prevail in $t = 2$ for those holding out in $t = 1$. Further, bailout recipients must be able to finance in $t = 2$, which requires (8) as well as $\theta_g \le \theta_0^*$. In particular, this condition requires that $S \ge \underline{S}_0$. Finally, we must also have $m(\theta_1, \gamma(\theta_1)) \ge p_g$, or else type θ_1 (and nearby types) would rather collect p_g in $t = 1$ and consume its asset in $t = 2$.

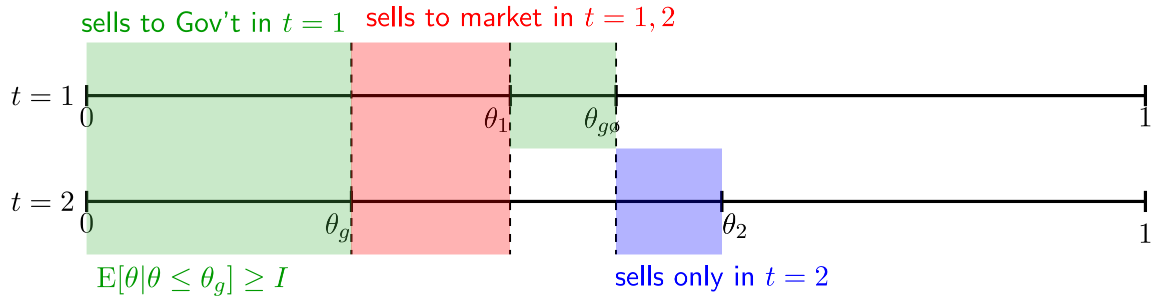


Figure 7 – Bailout Equilibrium with Market Rejuvenation (MR2)

the one-shot model.

For some other p_g's, a bailout can also attract high type firms that accept the bailout in $t=1$ but choose not to sell in $t=2$ (instead of being subject to the bailout stigma). Specifically, this equilibrium admits additional cutoff $\theta_{g\o} > \theta_1$, such that those with $\theta \in (\theta_1, \theta_{g\o})$ accept bailout in $t=1$ but never sell their remaining assets in $t=2$. This equilibrium, labeled **MR2**, is depicted in Figure 7.[29]

Remarkably, the types of firms accepting the government bailout are non-contiguous in this equilibrium. The reason for this is again the stigma associated with bailout. For p_g high enough, bailout becomes attractive to firms, but the bailout stigma remains a problem for them. This induces high type firms to receive bailout but never seek to sell assets in $t=2$. Consequently, unlike Tirole (2012), "dregs skimming" need not be the role of a bailout in the presence of the stigma.

For ease of analysis, we extend the regularity conditions in Assumption 1 to the following ones:

Assumption 2. *(i) For every* $0 < \tilde{\theta} < \theta < 1$, $\Delta(\theta; \tilde{\theta}, S) := m(\tilde{\theta}, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(0, \theta))$ *is decreasing in* θ*; (ii) If* $\Delta(0; \tilde{\theta}, S) \geq 0$*, then* $\Delta(0; \tilde{\theta}, S') \geq 0$ *for every* $S' > S$*; (iii) For every* $0 < \tilde{\theta} < \theta < 1$, $2m(\tilde{\theta}, \theta) - m(0, \tilde{\theta})$ *is decreasing in* $\tilde{\theta}$.

[29]This equilibrium is characterized by a number of conditions. First, conditions (10), (8), and $\theta_g \leq \theta_0^*$ continue to hold. But condition (11) is now replaced by indifference conditions for types θ_1 and $\theta_{g\o}$, which are given respectively as:

$$2m(\theta_g, \theta_1) + 2S = \theta_1 + p_g + S, \tag{12}$$
$$p_g + S + \theta_1 = \theta_1 + m(\theta_{g\o}, \gamma(\theta_{g\o})) + S, \tag{13}$$

and

$$p_g + S + \theta_{g\o} = \theta_{g\o} + m(\theta_{g\o}, \gamma(\theta_{g\o})) + S. \tag{14}$$

Like Assumption 1, Assumption 2 is also satisfied by the standard distribution functions commonly used in economic analysis. We now present the existence result for the MR-type equilibria.

Proposition 5. *(i) If $S \geq \underline{S}_0$, then there exist $\underline{p}_g^{MR} < \overline{p}_g^{MR}$ such that either the MR1 or the MR2 equilibrium, but not both, exists for each $p_g \in [\underline{p}_g^{MR}, \overline{p}_g^{MR})$.*

(ii) The total volume of asset trade is greater under the MR1 or MR2 equilibria than without bailouts if and only if $p_g \in [\breve{p}_g^{MR}, \overline{p}_g^{MR})$, where $\breve{p}_g^{MR} \in [\underline{p}_g^{MR}, \overline{p}_g^{MR}]$;

(iii) For any bailout equilibrium with market rejuvenation, there exists a BMS equilibrium that induces a larger total trade.

Proof. See Appendix A. *Q.E.D.*

A couple of findings are noteworthy. The first one is Proposition 5-(ii): a relatively low bailout term may lead to less trade than without bailout. We can observe this phenomenon in the MR2 equilibrium arising from relatively low bailout terms, where the bailout non-contiguously attracts high types which do not sell their assets in $t = 2$ due to the attached stigma. In equilibrium, the firms with sales to the market in $t = 1$ are deemed as those worse than the bailout recipients with high types and the holdout types in $t = 1$. A low bailout term further worsens the overall perception of those firms with sales to the market, which diminishes the overall trade to being lower than without bailout. However, we do not have this observation in the MR1 equilibrium, where the bailout solely plays its dregs-skimming role like in the static model of Tirole (2012).

The following example can facilitate to understand the feature aforementioned. Suppose $S > \underline{S}^*$ and there exists the MR2 equilibrium for a bailout term $p_g = m(\theta_1^*, \gamma(\theta_1^*))$, where θ_1^* is the cutoff type to sell in both periods in the equilibrium without bailout. In this equilibrium, the highest type to receive the bailout but not to sell in $t = 2$ is $\theta_{g\emptyset} := \theta_{g\emptyset}^{MR2} = \theta_1^*$ following from (14). Also, θ_1^{MR2} – the value of the marginal type with sales to the market in both periods – must be strictly lower than $\theta_{g\emptyset}^{MR2}$, which in turn means that $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\emptyset}^{MR2}))$, the total volume of trade in the MR2 equilibrium, is strictly less than $F(\theta_1^*) + F(\gamma(\theta_1^*))$, the total volume of trade in the equilibrium without bailout. Meanwhile, every MR1 equilibrium always yields more trade than without bailout: by Assumption 2, the highest type to sell in $t = 1$ in MR1 equilibrium is $\theta_1^{MR1} > \theta_1^*$; the total volume of trade in the MR1 equilibrium $F(\theta_1^{MR1}) + F(\gamma(\theta_1^{MR1}))$ is strictly greater than that in the equilibrium without bailout, $F(\theta_1^*) + F(\gamma(\theta_1^*))$.

The second finding, and perhaps the most striking one, is Proposition 5-(iii): market revival lowers the volume of trade sustainable by a given level of bailout. The reason is attributed to the cream-skimming effect of the market; the market attracts relatively high quality types, which aggravates the stigma placed on the bailout recipients. In equilibrium, the marginal type must be indifferent between accepting bailout and selling to the market, which in turn means that the perception of the market participants cannot be too favorable, either. In short, the cream skimming by the market brings down the overall perception of the firms selling their assets, and the overall trade, in $t = 1$ (as well as $t = 2$).

This last point can be argued more clearly by the following thought experiment. Suppose the highest type selling its asset (either to the market or to the government) in $t = 1$ is the same under both equilibria and equals $\hat{\theta} := \theta_1^{MR1} = \theta_g^{BMS}$. The marginal type's payoff from not selling is the same in both the MR1 and BMS equilibrium (and equals $\hat{\theta} + m(\hat{\theta}, \gamma(\hat{\theta})) + S$). But the marginal type's payoff from selling is different in the two equilibria. In the BMS equilibrium, the payoff is $p_g + m(0, \hat{\theta}) + 2S$ but in the MR1 equilibrium, it is $p_g + m(0, \theta_g^{MR1}) + 2S$. Since $\theta_g^{MR1} < \hat{\theta} = \theta_1^{MR1} = \theta_g^{BMS}$, the same marginal type would have strictly lower incentive to sell (to the market) in the MR1 equilibrium than (to sell to the government) in the BMS equilibrium. This reasoning suggests that the marginal type of sale is lower in the MR1 equilibrium.

4.2 Illustration via Example

Here we use a numerical example to highlight the main implications of the equilibria we identified. Suppose $I = \frac{1}{6} + \varepsilon$ for an arbitrarily small number $\varepsilon > 0$, $S = \frac{1}{3}$, and F is uniform on $[0, 1]$. In Tirole (2012)'s one-period model, absent bailout, the firms with types $\theta < \theta_0^* = \frac{2}{3}$ sell their assets at price $p_0^* = 1/3$. If the government offers bailout at $p_g > p_0^* = 1/3$, this enlists the market to offer the same price; and firms with $\theta \leq p_g - \frac{1}{3}$ accept the bailout and the firms with $\theta \in [p_g - \frac{1}{3}, (p_g + \frac{1}{3}) \wedge 1]$ sell to the market. In particular, $p_g = 2/3$ would eliminate adverse selection altogether and allow all firms to fund their projects.

In our two-period model, without government bailout, the market in $t = 1$ freezes completely due to the early sales stigma; and the market in $t = 2$ results in the equilibrium of Tirole (2012) without intervention. The effects of government bailouts under different p_g's are depicted in Figures 8. The left panel describes the volume of trade in $t = 1$ under alternative equilibria, and the right panel describes the volume of trade in $t = 2$. In each panel, the vertical axis depicts the level of bailout p_g, and the horizontal axis depicts the volume of trade, or the highest firm type that trades. Meanwhile, Figure 9 describes the improvement in total volume of trade across both periods over that without bailout, achieved under alternative equilibria.

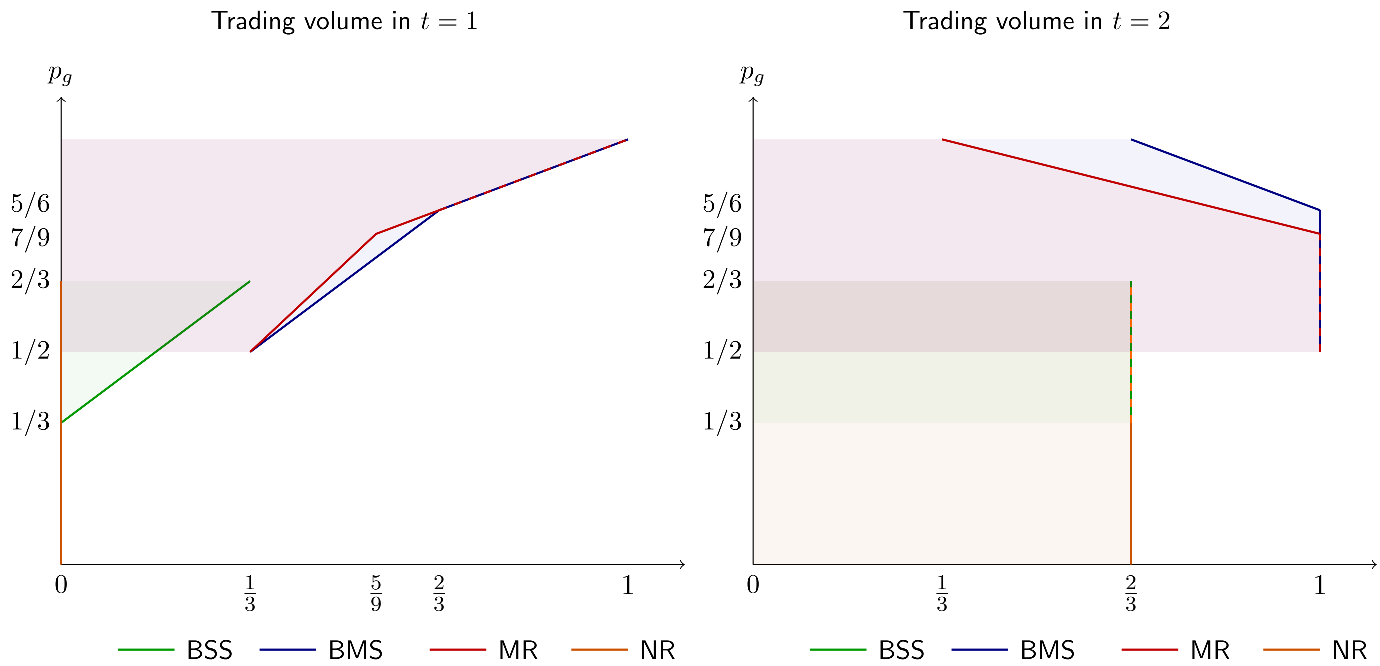


Figure 8 – Trading Volume in Alternative equilibria as a Function of p_g $(S = \frac{1}{3}, I = \frac{1}{6} + \varepsilon)$

Different equilibria exist for different ranges of p_g's. NR equilibrium exists for $p_g \in [0, 2/3]$; BSS equilibrium exists for $p_g \in [1/2, 2/3]$; BMS equilibrium exists for $p_g \geq 1/2$; MR1 equilibrium exists for $p_g \in [1/2, 7/9]$; and MR2 equilibrium exists for $p_g \geq 7/9$.

Several implications can be drawn from inspecting the equilibria.

☐ *Multiplicity of Equilibria:* The various manners in which participants may coordinate their actions and form beliefs lead to different outcomes. As seen in Figure 8, the BMS equilibrium coexists with the BSS equilibrium for $1/2 \leq p_g \leq 2/3$, and with the MR type equilibrium for $p_g \geq 1/2$. As explained following Proposition 4, the former multiplicity arises from the "self-fulfilling" nature of the bailout stigma—i.e., different ways beliefs may form on the bailout recipients on the equilibrium path. The latter multiplicity is the result of the endogeneity of the (out-of-equilibrium) belief for those selling to the market; a bailout at $p \geq 1/2$ can trigger (reasonably) unfavorable belief on the firms selling in the market to completely "crowd out" market sale, supporting the BMS equilibrium.

☐ *Response to "even an attractive" bailout can be lackluster:* As exemplified by Ford's refusal to accept the bailout offer quoted in the Introduction, distressed firms are notoriously reluctant to accept attractive bailout offers. This is most clearly captured by our "No Response" equilibrium in which a bailout offer $p_g \in [0, \frac{2}{3}]$ that should be attractive for firms with sufficiently low θ (irrespective of their investment decisions) is refused for fear of the bailout stigma. But the reluctance is exhibited in all other equilibria. Even offers $p_g \geq 2/3$ that are attractive enough to all types, and indeed eliminate adverse selection altogether in Tirole (2012) may be refused by some firms in all of the equilibria. Eliciting a clear response may require the bailout terms to be sufficiently attractive not just to low types but also to high types. For instance, an offer $p_g \geq 2/3$ can avoid both the NR and BSS type equilibria, thus guaranteeing a tangibly significant response.[30] This point is consistent with the so-called "money-in-the-window" wisdom—that an overwhelming response is required to avoid miscoordination of beliefs.

☐ *Improving the terms of bailouts can have a discontinuous effect:* Figure 8 shows that an increase in the bailout term p_g has a discontinuous reaction. For instance, for any bailout offer $p_g \in [1/3, 1/2]$ the only equilibrium is of the BSS-type, so the bailout elicits a low take-up. As p_g rises within $[1/3, 1/2]$, the uptake rate rises continuously. However, as p_g rises past 1/2, both the BMS and MR equilibria emerge. In fact, as p_g rises past 2/3, the BSS equilibrium disappears. Hence, regardless of how the equilibrium is selected, an increase in p_g will result in a discontinuous increase in the measure of firms accepting the bailout and funding their projects at some $p_g \in [1/2, 2/3]$.

☐ *Bailouts yield delayed and indirect benefits:* Lackluster responses to bailouts are often cited as evidence that the policies are ineffective. Our analysis challenges such a view. Even when the bailout has a low take-up, its mere presence could improve the outcome; firms that refuse the bailout boost their market perception sufficiently enough to improve their funding abilities later on. This is seen in Figure 9 by the fact that the increase in the net sale achieved for both periods exceeds that attained from $t = 1$ (left panel) by wide margin as p_g rises past the level that makes a tangible difference. That bailouts yield delayed benefits on those that "refuse" to accept them comprises a surprising but important takeaway lesson from the current paper. This point is consistent with the view that despite the initial lackluster response to the bailouts by the US government, the policy did make a difference.[31]

☐ *"Propping up" the market dampens overall investment:* With a bailout term $p_g \geq 1/2$, it is possible for the government to rejuvenate the private market, just as in Tirole's analysis, although the BMS type also remains an equilibrium. Comparison of the MR and BMS equilibria in Figure 9 reveals rather surprisingly that the total volume of trade supported by the MR

[30] Notice, however, that even such an offer does not elicit a full participation, as would have been the case with the one-shot model.

[31] Interestingly, the delayed benefit can be nonmonotonic in the size of bailout. In the right panel of Figure 8, the $t = 2$ trade declines as p_g rises above 7/9 in the MR2 equilibrium, and above 5/6 in the BMS equilibrium. The reason is that as p_g increases, more firms simply accept the bailout and choose not to sell in $t = 2$; namely, the "donut holes" in Figures 5 (bottom panel) and 7 expand as p_g increases (whereas θ_2 has reached its maximum of 1 and remains fixed). Note, however, the total trade (across the two periods) increases as p_g rises, as can be seen by Figure 9.

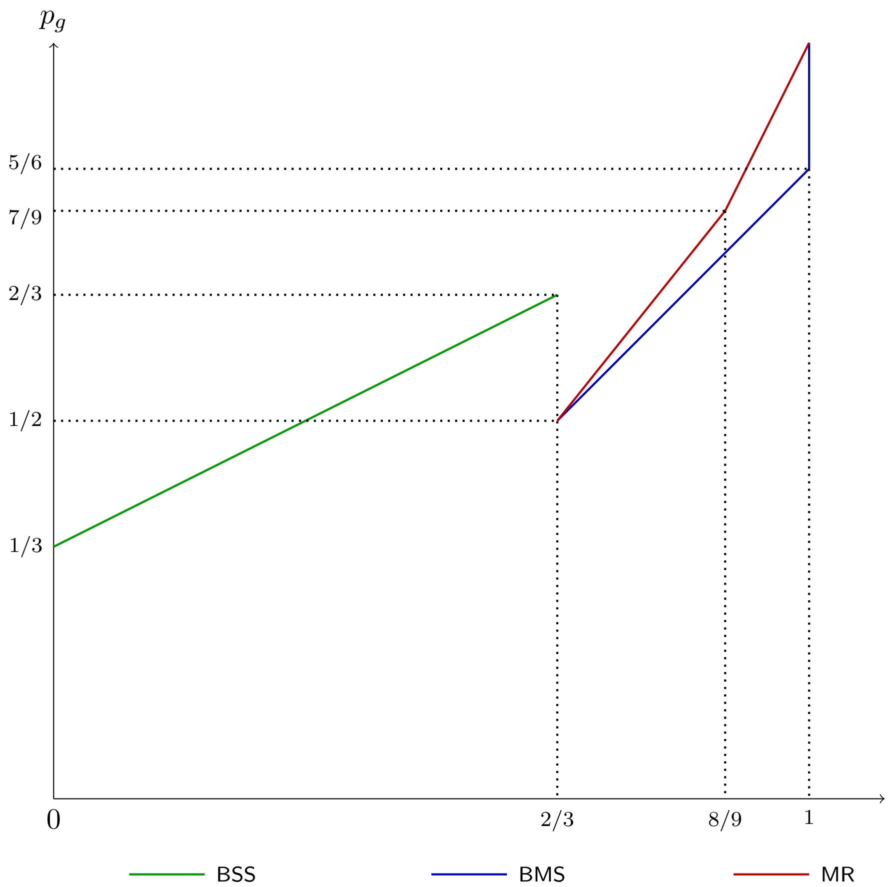


Figure 9 – Improvement in Trading Volume by Bailout in Alternative Equilibria ($S = \frac{1}{3}, I = \frac{1}{6} + \varepsilon$)

equilibrium is strictly less than that under the BMS equilibrium. As explained earlier, this is due to the "belief externality" a market option imposes on the recipients of bailouts: an active market in $t = 1$ worsens bailout stigma, which in turn brings down the perception of even those participating in the market. This is a surprising insight, which to our knowledge has never been recognized in the existing literature.

5 Secret Bailouts

A natural policy response to bailout stigma is to keep the identities of its recipients hidden from the market. Indeed, it is not uncommon for governments to offer protection of privacy for firms seeking financial bailout. For instance, the Federal Reserve conventionally runs the discount window as a measure to inject public liquidity to banks in need of short-term funding. The so-called discount window stigma is generated by the fact that borrowing banks can be easily identified because they no longer use the federal funds market, an alternative that banks usually rely on for short-term funding.[32] To reduce the stigma attached to the discount window, the Federal Reserve created the Term Auction Facility that is intended to conceal the identities of borrowing banks.

To examine the implication of such a **secret bailout**, we consider the same model as before with the government running the asset purchase program at price $p_g \geq I$ in $t = 1$, but suppose the firm's decision whether to accept the bailout is not observable to investors in the market.[33] For comparison, we call the asset purchase program in the previous section **transparent bailout**.

Under secrecy, the market cannot distinguish between the firms accepting bailouts and those that do not sell their assets in $t = 1$. The market observes only the set of firms selling assets to the market in $t = 1$. Thus secrecy allows the bailout recipients to pool with the holdouts. Since the latter is likely to be of high types, secrecy mitigates the stigma the bailout recipients suffer from under transparent bailout. In fact, one immediate effect is to eliminate "no uptake" equilibrium.

Lemma 3. *In equilibrium, a secret bailout with $p_g \geq \max\{I, p_1^*\}$, where p_1^* is the $t = 1$ market price absent government offer,[34] must be accepted by a positive measure of firms.*

Proof. See Appendix A. *Q.E.D.*

Lemma 3 means that any serious bailout offer must be accepted by some firms in equilibrium. Another important difference between transparent and secret bailouts is that, under the latter, the opportunity to boost reputation by delaying trade until $t = 2$ no longer exists. This is because the $t = 1$ holdouts are pooled together with the bailout recipients.

Lemma 4. *Under secret bailouts, there is no equilibrium in which the market is rejuvenated in $t = 1$ and some firms sell their assets only in $t = 2$.*

Proof. See Appendix A. *Q.E.D.*

[32] The Dodd-Frank Wall Street Reform and Consumer Protection Act passed in 2010 further requires the Federal Reserve to publicly disclose, with a two-year lag, the names of banks borrowing from the discount window and the total amount of money they borrow. This regulation will further facilitate identification of the firms using the discount window.

[33] One may question whether secret bailouts of this kind can be implemented without political influence or cronyism. Legislation is one way to tackle such problems. For example, a special act such as the Emergency Economic Stabilization Act can explicitly incorporate a clause that guarantees the disclosure of information at the end of the program and criminal liabilities for those who are found to have been involved in any wrongdoing.

[34] Let $p_1^* = 0$ in case the market freezes in $t = 1$ without any government intervention.

5.1 No Immediate Market Rejuvenation (SNS)

Secret bailouts may also fail to rejuvenate the market in $t = 1$. In this equilibrium, labeled SNS, the bailout recipient faces no stigma. The firm's equilibrium actions are depicted in the top panel of Figure 10. As before, a market sale in $t = 1$ is crowded out by bailout: no private buyer makes an offer since either it is all rejected or accepted only by sufficiently low types to result in loss to the buyer, all supported by the off-the-path belief that the firms accepting the (deviation) offer are of the worst type. Given this, the secrecy of bailout means that no updating of belief occurs in $t = 2$. It then follows that the $t = 2$ outcome is the same as in Tirole's one-period model, regardless of the firms' decisions in $t = 1$. Since there is no belief updating the first period decision is a myopic optimal one, a type θ-firm accepts the bailout if and only if $\theta \leq p_g + S$.

Proposition 6. *Under secrecy, if $p_g \geq I$, then there is an equilibrium in which (i) in $t = 1$, the market freezes and the firm accepts the government offer p_g if and only if $\theta \leq \min\{p_g + S, 1\}$; and (ii) in $t = 2$, the firm sells its asset at price $p_0^* = m(0, \theta_0^*)$ if and only if $\theta \leq \theta_0^*$.*

While a more precise comparison with transparent bailouts will be established in the next section, it is useful to observe the tradeoffs associated with secrecy. The obvious benefit from secrecy is the removal of stigma. This encourages participation, increasing the uptake rate of bailout, and stimulates investment in $t = 1$. The downside of removing bailout stigma is the elimination of the opportunity for high-type firms to signal their types by holding out in $t = 1$ and trade at a more favorable terms in $t = 2$. This means that secrecy will reduce the second-period trade relative to transparency.

5.2 Immediate Market Rejuvenation (SMR)

A secret bailout may also lead to an immediate market rejuvenation, i.e, an MR type equilibrium, labeled SMR. The firm's equilibrium actions in an MR equilibrium are depicted in the bottom panel of Figure 10, and characterized as follows.

Proposition 7. *In any equilibrium in which a secret bailout leads to an immediate market rejuvenation, there are cutoffs $0 < \theta_g < \theta_1 < \theta_{g\emptyset}$ such that (i) the types $\theta \leq \theta_g$ accept the government offer p_g in $t = 1$, and sell at price $m(0, \theta_g)$ in $t = 2$; (ii) the types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods; and (iii) the types $\theta \in (\theta_1, \theta_{g\emptyset}]$ accept the government offer but do not sell in $t = 2$. Such an equilibrium exists for a bailout offer $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$ for some $\overline{p}_g^{SMR} > \underline{p}_g^{SMR} \geq I$.*

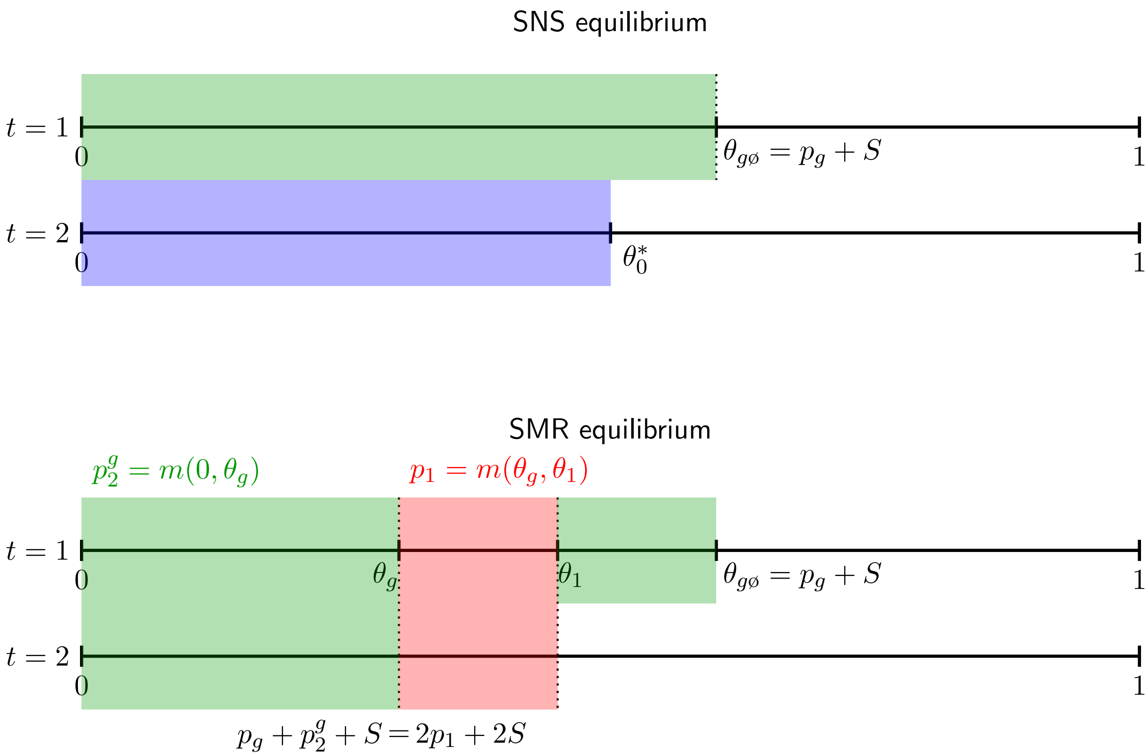


Figure 10 – The Bailout Equilibria under Secrecy.

Proof. See Appendix A. *Q.E.D.*

In this equilibrium, similar to the case of transparency, lowest types accept the bailout in $t = 1$ and sell to the market in $t = 2$ at a low price that reflects stigma, intermediate types sell to the market in both periods at a higher price, and higher types accept bailout but boycott the market altogether in $t = 2$.[35] Unlike transparent bailouts, no firms sell only in $t = 2$, as was shown previously in Lemma 4.

The most striking feature of this equilibrium is that secrecy does not eliminate bailout stigma. This is due both to the endogeneity of stigma and to the early market revival. True to its intent, secrecy protects the identities of the bailout recipients, in particular keeping them

[35] Critical types satisfy relevant indifference conditions. First, type θ_g is indifferent between between accepting the government offer and selling to the market in $t = 2$, or selling to the market in both periods. This indifference implies

$$p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1).$$

Next, type θ_1 is indifferent between selling to the market in both periods and accepting bailout in $t = 1$ but not selling in $t = 2$, which implies $2m(\theta_g, \theta_1) + 2S = p_g + S + \theta_1$. Combining the two indifference conditions leads to

$$\theta_1 = m(0, \theta_g) + S.$$

indistinguishable from the holdouts. Nevertheless, differing "willingness" to sell in $t = 2$ reveals their types, those willing to sell being perceived as low types. Of course, this is a core feature of the lemons markets, which is present even in the SNS equilibrium; but in that context, the feature entails no worse selection in $t = 2$ than in the one-period lemons model. In the current setting, however, the presence of the types selling to the market in $t = 1$ entails a severe stigma, since those that do not sell to the market in $t = 1$ and are seeking to sell in $t = 2$ are revealed to be worse than those that sold to the $t = 1$ market. The flip side of the problem is that the high types $[\theta_1, \theta_{g\o}]$ obtaining the bailout boycott the $t = 2$ market to avoid the stigma, which in turn makes the stigma severe.

The stigmatizing effect of early market revival has a significant effect on the overall trading and investment activities. As discussed earlier, the inactive private market in $t = 1$ in the SNS equilibrium effectively eliminates both the early sales stigma and the bailout stigma. This leads to a lower trade volume and investment in the SMR equilibrium than in the SNS equilibrium, given the same government offer p_g.

Proposition 8. *Suppose a secret bailout admit an SMR equilibrium. Then, the same bailout admits an SNS equilibrium that induces a (weakly) higher total trade volume.*

Proof. See Appendix A. *Q.E.D.*

6 Welfare and Optimal Policy Design

The preceding analysis has examined how bailout stigma affects the outcomes arising from a variety of bailout scenarios. We now turn to their welfare implications and the design of optimal bailout policy. For a fuller analysis of welfare, we also need to consider the cost of providing bailout, including the cost of raising public fund. In keeping with the standard approach as well as Tirole (2012), we assume that raising a dollar of public fund entails a deadweight loss of $\lambda > 0$.

As will be seen, both welfare and policy analyses are greatly facilitated by recasting the bailout problem in a mechanism design framework. This latter framework would allow for a general class of bailout policies that the government may employ. For instance, our framework allows for a menu of bailout packages that differ in the bailout terms and disclosure options, possibly revealing the identities of firms choosing one package but concealing the identities of the firms choosing a different package.

The two important restrictions we shall impose are that (1) the government never offers a stochastic policy and never rations a firm on the offered package and that (2) the government offers no bailout and never intervenes in the market in $t = 2$. The second assumption accords well with the empirical fact—as well as being consistent with the treatment so far—that the bailout is often confined to a limited duration (modeled in our paper by $t = 1$).[36] The first assumption is also realistic, consistent with government practices.

To begin, we first appeal to the revelation principle and view the government as proposing a mechanism that specifies the quantity $q(\theta) \in \mathcal{Q} := \{0, 1, 2\}$ of the asset the firms sell and the transfer $t(\theta) \in \mathbb{R}$ the firms receive, as a function of their reported types θ. The restriction to deterministic allocation stems from the restriction (1) above. The resulting map $(q, t) : [0, 1] \to \mathcal{Q} \times \mathbb{R}$ then describes a **mechanism** or equivalently an **outcome** that may arise in an equilibrium under a policy treatment. Since the only reason for selling an asset for a firm is to finance the project and enjoy the surplus, it is without loss to restrict attention to mechanisms in which $t(\theta) \geq I$. For any mechanism $M = (q, t)$, if a firm with type θ reports $\tilde{\theta}$, it gets the payoff

$$U^M(\tilde{\theta}|\theta) := t(\tilde{\theta}) + \theta(2 - q(\tilde{\theta})) + Sq(\tilde{\theta}),$$

since each unit of asset sold enables the financing of a unit of project with net surplus S and the remaining unsold units $(2 - q(\tilde{\theta}))$ yields the value θ to the firm. Of course, for any outcome $M = (q, t)$ to be consistent with equilibrium, it must be incentive compatible:

$$u^M(\theta) := U^M(\theta|\theta) \geq U^M(\tilde{\theta}|\theta) \;\; \forall \theta, \tilde{\theta} \in [0, 1]. \qquad (IC)$$

Next, each firm has the option of not participating and enjoying the payoff realized from its asset. In other words,

$$u^M(\theta) \geq 2\theta \;\; \forall \theta, \tilde{\theta} \in [0, 1]. \qquad (IR)$$

Since a dollar deficit entails a deadweight loss of $\lambda > 0$, the social welfare from a mechanism $M = (q, t)$ is given by:

$$W(M) := \int_0^1 \left[u^M(\theta) + \theta q(\theta) - t(\theta) - \lambda(t(\theta) - \theta q(\theta)) \right] f(\theta) d\theta,$$

where the first term is the surplus accruing to the firms, the next two terms $\theta q(\theta) - t(\theta)$ aggregate the surplus the government and the market enjoy, and finally the last terms $\lambda(t(\theta) - \theta q(\theta))$ account for the deadweight loss associated with deficit the government runs (and thus must finance through distortionary measures). Note that market must break even in any equilibrium,

[36] As mentioned early, governments are reluctant to enter into protracted and large scale bailouts that can be seen effectively as nationalizing private firms.

so the government may need to bear net deficit to support asset trade.

In the sequel, we are interested in the case in which, absent government bailout, not all types of firm can sell to the competitive market. Due to the cost of public fund ($\lambda > 0$), the government will not wish to offer strict rents to the highest type. That is, (IR) is binding for type $\theta = 1$ firm: type $\theta = 1$ firm enjoys the payoff of 2 if it fails to sell its asset or, even if it sells the asset, it will never receive the payment greater than the maximum possible value. As will be seen, given these two conditions, (IR) will hold for all other types of firms. Finally, the government in our model offers bailout only in $t = 1$ and never intervenes in the $t = 2$ market. This constraint, along with the availability of the market willing to purchase assets at break-even price, must limit the set of possible outcomes. We let $\mathcal{M}$ denote the set of all mechanisms/outcomes that satisfy all these properties. The next lemma provides a characterization on how the features of our model and the basic feasibility limit the set of implementable allocations:

Lemma 5. *We have the following observations:*

(i) If $M = (q, t) \in \mathcal{M}$, then $q(\cdot)$ is nonincreasing, and $q(\theta) \geq 1$ for all $\theta \leq \theta_0^*$ and $q(\theta) \leq 1$ for all $\theta > \theta_0^*$, where θ_0^* is the highest type that sells asset in the one-shot model without government bailout.

(ii) [Revenue Equivalence] If $M = (q, t)$ and $M' = (q', t')$ both in $\mathcal{M}$ have $q = q'$, then $W(M) = W(M')$. In other words, an equilibrium allocation pins down the welfare, expressed as follows:

$$\int_0^1 \left[J(\theta)q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta, \tag{15}$$

where

$$J(\theta) := (1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}.$$

(iii) Consider two possible equilibria, labeled A and B, (possibly associated with different levels of p_g or by different disclosure policies) such that equilibrium $i = A, B$ induces trade volume $q_i(\cdot)$ across the two periods. Suppose

$$\int_0^1 q_A(\theta)f(\theta)d\theta = \int_0^1 q_B(\theta)f(\theta)d\theta$$

but there exists $\tilde{\theta} \in (0, 1)$ such that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$. Then, equilibrium A yields higher welfare than equilibrium B, strictly so if $q_A(\theta) \neq q_B(\theta)$ for a positive measure of θ's.

Proof. See Appendix A. *Q.E.D.*

Part (i) of Lemma 5 characterizes the set of possible allocations that are consistent with incentive compatibility and the government's laissez-faire approach in $t = 2$. In particular, it implies that no firm with type greater than the one-shot threshold can sell in both periods and no firm with type less than the one-shot threshold will fail to sell even one unit. While this characterization involves a nontrivial restriction on the set of possible allocations, it is worth emphasizing that it allows for a very general set of bailout policies that the government may employ in terms of the bailout terms and disclosure options. For instance, the government may offer a menu of bailout packages with varying degrees of disclosure. Formally, the government may offer a menu of packages $\{(p_g^i, \gamma^i)\}_{i \in I}$, where I is an arbitrary index set, such that a set of firms choosing package i is allowed to sell its asset in $t = 1$ at price p_g^i and their identities are revealed with probability $\gamma^i \in \{0, 1\}$.[37] One simple example is that the government offers a menu of two packages $\{(p_g^1, 1), (p_g^2, 0)\}$ so that those firms that pick the first package can sell their assets in $t = 1$ at price p_g^1, which is revealed to the market, and those that choose the second package can sell their assets in $t = 1$ at price p_g^2, and the identities of these firms are concealed from the market. Our framework will encompass all such possibilities; in short, we shall allow for arbitrary bailout and disclosure policies the government may employ.

Part (ii) identifies the social value associated with firms' asset trading. Specifically, the sale of type θ-firm's asset generates the virtual value $J(\theta) = (1 + \lambda)S - \lambda \frac{F(\theta)}{f(\theta)}$. This consists of two parts: The first is the social value $(1 + \lambda)S$ of inducing a sale, namely the value of funding the investment project. The second is the deadweight loss $\lambda \frac{F(\theta)}{f(\theta)}$ required to incentivize the sale. The incentive cost is increasing in θ since a higher type firm requires stronger incentive to sell.

Part (iii) of Lemma 5 suggests that all else equal, the welfare will be enhanced when the trade cutoffs are equalized across the two periods. The simple intuition has to do with the fact that the incentive cost of enabling trade is higher for a higher-type firm; equalizing the cutoff types minimize the incentive costs. This observation facilitates comparisons of alternative equilibria studied in the previous sections. To economize on description, we say that an equilibrium of type A **dominates in welfare** an equilibrium of type B, *if for any equilibrium with property B that arises under any bailout term p_g, there is an equilibrium with property A arising from some bailout term p_g' which yields a weakly higher welfare, and strictly higher welfare in some instance.*

Proposition 9. *The equilibria are compared as follows.*

[37] For instance, the government induces a set of types given by a subdistribution F^i of F, where $0 \leq F^i(\theta) \leq F(\theta)$ for all θ, and both F^i and $1 - F^i$ are nondecreasing.

(i) Given a transparent bailout policy, an equilibrium with no market rejuvenation dominates in welfare an equilibrium with market rejuvenation.

(ii) Given a secret bailout policy, an equilibrium with no market rejuvenation dominates in welfare an equilibrium with market rejuvenation.

(iii) With or without market rejuvenation, an equilibrium under secrecy dominates in welfare an equilibrium under transparency.

Proof. See Appendix A. *Q.E.D.*

The proposition suggests that secrecy improves welfare relative to transparency, whether it is accompanied with immediate market revival or or not. As noted before, a secret bailout mitigates stigma for bailout recipients and this increase uptake in $t = 1$ but removes the opportunity for firms to build reputation via holdout and sell at more favorable terms in $t = 2$. Hence, it increases asset sale in $t = 1$ but reduces sale in $t = 2$ compared with a transparent bailout. Since there is too little trade in general, this has an equalizing effect in terms of the trade decision across the two periods. As observed, this improves welfare.

Second, the early revival of market hurts welfare. The reason is that early market revival exacerbates bailout stigma in both transparent and secret bailouts. This creates further wedge between the trades implemented across the two periods, and according to Part (iii) of Lemma 5, this is not a good thing from the welfare perspective.

Combining the results, secret bailout without immediate market revival comes out the best among the equilibria studied so far. Indeed, one can show the regime to implement a (constrained) optimum among the full set of outcomes. To show this formally, we use Parts (i) and (ii) of Lemma 5 to formulate the following relaxed program:

$$[P] \qquad \max_{q:[0,1]\to\mathcal{Q}} \int_0^1 \left[J(\theta)q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta$$

subject to

$$q(\cdot) \text{ is nondecreasing;}$$

$$q(\theta) \geq 1 \text{ if } \theta < \theta_0^* \text{ and } q(\theta) \leq 1 \text{ if } \theta > \theta_0^*.$$

To characterize the optimal solution, note first that the virtual value $J(\theta)$ is decreasing in θ, given the log-concavity assumption. We can thus define a cutoff type

$$\hat{\theta}^* := \sup \left\{ \theta \in [0,1] \middle| (1+\lambda)S \geq \lambda \frac{F(\theta)}{f(\theta)} \right\}.$$

Proposition 10. *The optimal bailout mechanism has*

$$q^*(\theta) = \begin{cases} 2 & \text{if } \theta \leq \min\{\hat{\theta}^*, \theta_0^*\} \\ 1 & \text{if } \theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] \\ 0 & \text{if } \theta > \max\{\hat{\theta}^*, \theta_0^*\}. \end{cases}$$

The optimal policy is implemented by a secret bailout policy in $t = 1$ *with* $p_g = \hat{\theta}^* - S$ *via an equilibrium that involves no immediate market rejuvenation.*

Proof. See Appendix A. *Q.E.D.*

7 Conclusion

This paper has studied a dynamic model of government bailout to analyze how stigma affects participation and the costs of bailout. We briefly recapitulate our main findings. First, compared to the static setting without reputational concern, the private market is more likely to freeze in the dynamic setting. Second, government bailout under transparency leads to multiple equilibria: in the first type of equilibria, otherwise attractive bailout offers have no effect; in the second type, bailout offers cannot immediately rejuvenate the private market, hence the government crowds out the private market in the short term; in the third type, the market can be immediately rejuvenated, but the cost it too high so that such equilibria are dominated by the second type. Third, secret bailouts can alleviate the concern for bailout stigma and are shown to dominate transparent bailouts.

The central lesson from the current work is that, compared to the static setting, the efficacy of bailout needs more careful assessment in the dynamic setting due to the interplay between the bailout stigma and early sales stigma, and the market's belief on and off the equilibrium path. As we have shown in our two period model, bailout offers that would be effective in the static setting may have no effect and bailout may not even play the dregs skimming role in the dynamic setting. To the extent that many participants in public bailout programs are long-lived financial institutions, the concern about stigma needs to be explicitly factored into policy discussions. Our analysis of secret bailouts and optimal policy design is a moderate first step in this direction. Needless to say, more work needs to be done as to how optimal policies can be implemented in practice.

References

Allen, Franklin, and Douglas Gale. 2000. "Financial Contagion." *Journal of Political Economy*, 108(1): 1–33.

Armantier, Olivier, Eric Ghysels, Asani Sarkar, and Jeffrey Shrader. 2011. "Stigma in Financial Markets: Evidence from Liquidity Auctions and Discount Window Borrowing during the Crisis." Federal Reserve Bank of New York Staff Report No. 483.

Bagnoli, Mark, and Ted Bergstrom. 2005. "Log-Concave Probability and its Applications." *Economic Theory*, 26(2): 445–469.

Bernanke, Ben S. 2009. "The Federal Reserve's Balance Sheet." Speech Given on April 3, 2009 at the Federal Reserve Bank of Richmond 2009 Credit Markets Symposium, Charlotte, North Carolina.

Camargo, Braz, and Benjamin Lester. 2014. "Trading Dynamics in Decentralized Markets with Adverse Selection." *Journal of Economic Theory*, 153: 534–568.

Cassola, Nuno, Ali Hortaçsu, and Jakub Kastl. 2013. "The 2007 Subprime Market Crisis through the Lens of European Central Bank Auctions for Short-Term Funds." *Econometrica*, 81(4): 1309–1345.

Chari, VV, Ali Shourideh, and Ariel Zetlin-Jones. 2014. "Reputation and Persistence of Adverse Selection in Secondary Loan Markets." *American Economic Review*, 104(12): 4027–4070.

Congressional Budget Office. 2012. "Report on the Troubled Asset Relief Program - March 2012."

Corbett, Jenny, and Janet Mitchell. 2000. "Banking Crises and Bank Rescues: The Effect of Reputation." *Journal of Money, Credit and Banking*, 474–512.

Daley, Brendan, and Brett Green. 2012. "Waiting for News in the Market for Lemons." *Econometrica*, 80(4): 1433–1504.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 401–419.

Diamond, Douglas W, and Raghuram G Rajan. 2005. "Liquidity Shortages and Banking Crises." *The Journal of Finance*, 60(2): 615–647.

Ennis, Huberto M, and John A Weinberg. 2013. "Over-the-Counter Loans, Adverse selection, and Stigma in the Interbank Market." *Review of Economic Dynamics*, 16(4): 601–616.

Fleming, Michael J. 2012. "Federal Reserve Liquidity Provision during the Financial Crisis of 2007-2009." Federal Reserve Bank of New York Staff Report No. 563.

Fuchs, William, and Andrzej Skrzypacz. 2015. "Government Interventions in a Dynamic Market with Adverse Selection." *Journal of Economic Theory*, 158: 371–406.

Fuchs, William, Aniko Öry, and Andrzej Skrzypacz. 2012. "Transparency and Distressed Sales under Asymmetric Information." *mimeo.* University of California, Berkeley.

Fudenberg, Drew, and Jean Tirole. 1991. "Game theory. 1991." *Cambridge, Massachusetts*, 393.

Furfine, Craig. 2001. "The Reluctance to Borrow from the Fed." *Economics Letters*, 72(2): 209–213.

Furfine, Craig. 2003. "Standing Facilities and Interbank Borrowing: Evidence from the Federal Reserve's New Discount Window." *International Finance*, 6(3): 329–347.

Gauthier, Céline, Alfred Lehar, Héctor Pérez Saiz, Moez Souissi, et al. 2015. "Emergency Liquidity Facilities, Signalling and Funding Costs." Bank of Canada Staff Working Paper 2015-44.

Hörner, Johannes, and Nicolas Vieille. 2009. "Public vs. Private Offers in the Market for Lemons." *Econometrica*, 77(1): 29–69.

Hoshi, Takeo, and Anil K Kashyap. 2010. "Will the US Bank Recapitalization Succeed? Eight Lessons from Japan." *Journal of Financial Economics*, 97(3): 398–417.

Inderst, Roman, and Holger M Müller. 2002. "Competitive Search Markets for Durable Goods." *Economic Theory*, 19(3): 599–622.

Janssen, Maarten CW, and Santanu Roy. 2002. "Dynamic Trading in a Durable Good Market with Asymmetric Information." *International Economic Review*, 257–282.

Journal of Economic Perspectives. 2015. "Symposia: the Bailouts of 2007 - 2009." *The Journal of Economic Perspectives*, 29(2): 3–122.

Kim, Kyungmin. 2012. "Information about Sellers? Past Behavior in the Market for Lemons." *mimeo.* University of Iowa.

Kremer, Ilan, and Andrzej Skrzypacz. 2007. "Dynamic Signaling and Market Breakdown." *Journal of Economic Theory*, 133(1): 58–82.

Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov. 2014. "Sizing Up Repo." *The Journal of Finance*, 69(6): 2381–2417.

Laeven, Luc, and Fabian Valencia. 2008. "Systemic Banking Crises: A New Database." IMF Working Paper 08/224.

La'O, Jennifer. 2014. "Predatory Trading, Stigma and the Fed? s Term Auction Facility." *Journal of Monetary Economics*, 65(C): 57–75.

Moreno, Diego Alejandro, and John Wooders. 2010. "Decentralized Trade Mitigates the Lemons Problem." *International Economic Review*, 51(2): 383–399.

Noldeke, Georg, and Eric Van Damme. 1990. "Signalling in a Dynamic Labour Market." *The Review of Economic Studies*, 57(1): 1–23.

Peristiani, Stavros. 1998. "The Growing Reluctance to Borrow at the Discount Window: An Empirical Investigation." *Review of Economics and Statistics*, 80(4): 611–620.

Philippon, Thomas, and Philipp Schnabl. 2013. "Efficient Recapitalization." *The Journal of Finance*, 68(1): 1–42.

Philippon, Thomas, and Vasiliki Skreta. 2012. "Optimal Interventions in Markets with Adverse Selection." *American Economic Review*, 102(1): 1–28.

Stern, Gary H, and Ron J Feldman. 2004. *Too Big to Fail: The Hazards of Bank Bailouts.* Brookings Institution Press.

Swinkels, Jeroen M. 1999. "Education signalling with preemptive offers." *The Review of Economic Studies*, 66(4): 949–970.

Szalay, Dezsö. 2012. "Strategic Information Transmission and Stochastic Orders." *mimeo.* University of Bonn.

Tirole, Jean. 2012. "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning." *American Economic Review*, 102(1): 29–59.

Veronesi, Pietro, and Luigi Zingales. 2010. "Paulson's Gift." *Journal of Financial Economics*, 97(3): 339–368.

Wilson, Linus, and Yan Wendy Wu. 2012. "Escaping TARP." *Journal of Financial Stability*, 8(1): 32–42.

A Appendix: Proofs

Proof of Lemma 1. Fix a game with discount factor $\delta \in (0,1)$ and fix an equilibrium of the required form. Let $(q_1(\theta), q_2(\theta))$ be the units of the asset a type-θ firm sells in each of the two periods and $(t_1(\theta), \theta_2(\theta))$ be the corresponding transfers.

Step 1. There exists $0 \leq \hat{\theta} \leq \check{\theta} \leq \tilde{\theta} \leq 1$ such that $q_1(\theta) = q_2(\theta) = 1$ for $\theta < \hat{\theta}$; $q_1(\theta) = 1, q_2(\theta) = 0$ for any $\theta \in (\hat{\theta}, \check{\theta})$; $q_1(\theta) = 0, q_2(\theta) = 1$ for any $\theta \in (\check{\theta}, \tilde{\theta})$; and $q_1(\theta) = q_2(\theta) = 0$ for any $\theta > \tilde{\theta}$.

Proof. In pure strategy equilibrium, we have $q_i(\theta) \in \{0, 1\}$ for each θ, $i = 1, 2$. The expected discounted payoff for type-θ when playing as type-θ' is $u(\theta'; \theta) := q_1(\theta')[S + t_1(\theta')] + [1 - q_1(\theta')]\theta + \delta[q_2(\theta')(S + t_2(\theta')) + (1 - q_2(\theta'))\theta]$. Let $Q(\cdot) := q_1(\cdot) + \delta q_2(\cdot)$ and $t(\cdot) := q_1(\cdot)t_1(\cdot) + \delta q_2(\cdot)t_2(\cdot)$. Since we have $u(\theta; \theta) \geq u(\theta'; \theta)$ for all θ, θ', it follows that $(S - \theta)[Q(\theta) - Q(\theta')] \geq t(\theta') - t(\theta)$. Similarly we must have $u(\theta'; \theta') \geq u(\theta; \theta')$, which leads to $(S - \theta')[Q(\theta') - Q(\theta)] \geq t(\theta) - t(\theta')$. Combining these two inequalities, we have $(\theta - \theta')[Q(\theta) - Q(\theta')] \leq 0$. This implies monotonicity: $Q(\theta) \leq Q(\theta')$ for any $\theta \geq \theta'$. Since $q_i(\theta) \in \{0, 1\}$ for each θ, $i = 1, 2$, the monotonicity implies the desired property.

Step 2. If $\check{\theta} < 1$, then those that hold out in $t = 1$ must be offered $m(\check{\theta}, \gamma(\check{\theta}))$ in equilibrium, which is accepted by types $\theta \in (\check{\theta}, \gamma(\check{\theta}))$ where $\gamma(\check{\theta}) > \check{\theta}$.

Proof. The belief in $t = 2$ for the holdouts is the truncated distribution of F on $[\check{\theta}, 1]$. This is essentially a one-shot problem with a truncated support. Thus the stated result follows from the definition of γ.

Step 3. $\hat{\theta} = \check{\theta}$.

Proof. Suppose to the contrary that $\hat{\theta} < \check{\theta}$. Let p be the price offered in $t = 1$. By the zero profit condition, the price p must break even with respect to the measure of types that accept it. Since type-$\check{\theta}$ firm must weakly prefer accepting p in $t = 1$ to not selling in either periods, we have

$$p + S + \delta\check{\theta} \geq (1 + \delta)\check{\theta} \Leftrightarrow p + S \geq \check{\theta}. \tag{16}$$

This means that all firms accepting p in $t = 1$ will accept the same price p in $t = 2$ if that price were offered in $t = 2$, with strict incentive for all firms with $\theta < \check{\theta}$. The fact that types $\theta \in (\hat{\theta}, \check{\theta})$ choose not to sell in $t = 2$ means that the price offered in $t = 2$ to those that accept p in $t = 1$, denoted by p^-, is strictly less than p.

Suppose first $\check{\theta} < 1$. Then, by Step 2, an offer $p_2 := m(\check{\theta}, \gamma(\check{\theta}))$ must be made in equilibrium to those that hold out in $t = 1$, which is accepted by types $\theta \in (\check{\theta}, \gamma(\check{\theta}))$. Since type-$\check{\theta}$ must be indifferent between selling at p in $t = 1$ only and selling at p_2 in $t = 2$ only, we must have

$$p + S + \delta\check{\theta} = \check{\theta} + \delta(p_2 + S). \tag{17}$$

In particular, $p_2 + S > \check{\theta}$, so $p + S > \check{\theta}$. This means that, if a buyer deviates by offering $p - \varepsilon > p^-$ for sufficiently small $\varepsilon > 0$ in $t = 2$ to those that accepted p in $t = 1$, then all of them must accept that offer. The buyer makes a strictly positive profit with such a deviation since p is the break-even price. We thus have a contradiction.

Suppose next $\check{\theta} = 1$, meaning that all types sell in $t = 1$. Here we invoke D1 refinement to derive a contradiction. In the candidate equilibrium, the payoff for type $\theta > \hat{\theta}$ is $p + S + \delta\theta$, and the payoff for all types $\theta < \hat{\theta}$ is some constant u^*, which must equal $p + S + \delta\hat{\theta}$ since type-$\hat{\theta}$ must be indifferent. Let $U^*(\theta)$ be the equilibrium payoff type-θ enjoys when the candidate equilibrium is played: $U^*(\theta) := \max\{u^*, p + S + \delta\theta\}$. When type-$\theta$ deviates by choosing holdout in $t = 1$, let p_2' be the market's offer in $t = 2$. Type-θ's deviation payoff is then $\theta + \delta \max\{p_2' + S, \theta\}$. Thus, when type-$\theta$ deviates by choosing holdout, the set of market's offers in $t = 2$ that dominate the candidate equilibrium for type-θ is

$$D(\text{holdout}, \theta) := \{p_2' | \theta + \delta \max\{p_2' + S, \theta\} \geq \max\{u^*, p + S + \delta\theta\}\}.$$

Note that for fixed p_2', the payoff difference $\theta + \delta \max\{p_2' + S, \theta\} - \max\{u^*, p + S + \delta\theta\}$ is strictly increasing in θ, so the set $D(\text{holdout}, \theta)$ is nested in the sense that $D(\text{holdout}, \theta) \subset D(\text{holdout}, \theta')$ for $\theta' > \theta$. In other words, $D(\text{holdout}, 1)$ is maximal, and more importantly, $D(\text{holdout}, \theta)$ is not maximal if $\theta < 1$. Given this, D1 refinement entails that the belief by the market must be supported on $\theta = 1$ in case of holdout. Thus following the deviation, the market's offer must be $p_2' = 1$. This means that, for the market's offer in the candidate equilibrium to satisfy D1, type $\check{\theta} = 1$ must enjoy the payoff of at least $1 + \delta(1 + S)$ in case of deviation to holdout from the candidate equilibrium. Since type $\check{\theta} = 1$ chooses to sell in $t = 1$ in the candidate equilibrium, we must have

$$p + S + \delta \geq 1 + \delta(1 + S), \tag{18}$$

which implies $p + S > 1$. This in turn implies that in $t = 2$, a buyer can deviate by offering a price slightly below p and induce acceptance from all types that accepted p in $t = 1$. Once again, the buyer makes a strictly positive profit with such a deviation, hence a contradiction.

Step 4. All types $\theta < \hat{\theta} = \check{\theta}$ (if is nonempty) are offered a single price in both periods equal to $m(0, \hat{\theta})$. All types $\theta > \hat{\theta}$ are offered price $m(\hat{\theta}, \gamma(\hat{\theta}))$, which is accepted by types $\theta \in (\hat{\theta}, \gamma(\hat{\theta}))$.

Proof. Suppose there are two distinct prices p, p' that are accepted by positive measures of firms. By the zero profit condition, both prices must break even with respect to the types that accept them. But then, no type will accept the lower price, hence a single price is offered to all types $\theta < \hat{\theta}$. The market's break-even condition then pins down the price at $m(0, \hat{\theta})$. The second statement follows from Step 2.

Step 5. If $\hat{\theta} = 1$, then $S \geq 2(1 - \mathrm{E}[\theta])$.

Proof. Applying the D1 argument as in Step 3, type-$\hat{\theta}$'s equilibrium payoff $(1+\delta)(p+S)$ should not be smaller than $1+\delta(1+S)$ where $p = m(0,1) = \mathrm{E}[\theta]$ by Step 4. From $(1+\delta)(p+S) \geq 1+\delta(1+S)$ follows the stated condition as $\delta \to 1$. *Q.E.D.*

Proof of Proposition 2. To prove (i), note first that the existence of cutoffs θ_1 and θ_2 was established by Lemma 1. Thus it suffices to show $\theta_1 \leq \theta_0 \leq \theta_2$ with strict inequalities if $\theta_0^* \in (0,1)$. Consider the $t = 1$ cutoff θ_1. If $\theta_1 < 1$, then it satisfies $\Delta(\theta_1, S) \leq 0$. Thus $\theta_1 \leq 2m(0,\theta_1) - m(\theta_1, \gamma(\theta_1)) + S < m(0,\theta_1) + S$ since $m(0,\theta_1) < m(\theta_1, \gamma(\theta_1))$. Since $\theta_0^* := \sup\{\theta | \theta \leq m(0,\theta) + S\}$, we have $\theta_1 \leq \theta_0^*$ with strict inequality if $\theta_0^* < 1$. If $\theta_1 = 1$, then $\Delta(\theta_1, S) \geq 0$ by D1 refinement, which in turn implies $\theta_0^* = 1$. Next, the $t = 2$ cutoff $\theta_2 = \gamma(\theta_1)$ satisfies $\theta_2 \leq m(\theta_1, \gamma(\theta_1)) + S$. Since $\theta_1 \geq 0$, we have $\theta_2 = \gamma(\theta_1) \geq \gamma(0) = \theta_0^*$, where the inequality is strict for $\theta_0^* < 1$ and $\theta_1 > 0$.

For (ii), note that, by Assumption 1, $\theta_1 - 2m(0,\theta_1) + m(\theta_1, \gamma(\theta_1))$ is strictly increasing in θ_1 for each S. Since $\theta_1 < 1$ satisfies $\Delta(\theta_1, S) \leq 0$, a unique θ_1 can be found, which is increasing in S. From this and the first claim follows the second claim.

For (iii), consider the candidate equilibrium in which the market in $t = 1$ completely freezes. This means that, in $t = 2$, we have one period equilibrium with cutoff given by θ_0^* and price p_0. In this equilibrium, the payoff for type $\theta \leq \theta_0^*$ is $\theta + \delta(p_0 + S) = \theta + \delta\theta_0^*$ and the payoff for type $\theta > \theta_0^*$ is $(1+\delta)\theta$. Thus the equilibrium payoff for type θ is $U^*(\theta) = \max\{\theta + \delta\theta_0^*, (1+\delta)\theta\}$. Suppose a buyer deviates and offers p_1 in $t = 1$. Let p_2 be the market's offer in $t = 2$ to those that accept the deviation offer p_1. Then the payoff to type θ from accepting p_1 is $p_1 + S + \delta \max\{p_2 + S, \theta\}$. As in the proof of Lemma 1, define the set

$$D(\text{sell}, \theta) := \{p_2 | p_1 + S + \delta \max\{p_2 + S, \theta\} \geq \max\{\theta + \delta\theta_0^*, (1+\delta)\theta\}\}.$$

For fixed p_2, the payoff difference is strictly decreasing in θ, hence $D(\text{sell}, 0)$ is maximal. Then by D1 refinement, the market's belief must be supported on $\theta = 0$ when a firm accepts a deviation offer p_1 in $t = 1$. Given this belief, no firm will accept p_1 if $p_1 \leq p_0$. If $p_1 > p_0$, then all types $\theta \leq p_1 + S$ accept the deviation offer p_1. Then the buyer will lose money given $p_1 > p_0$. *Q.E.D.*

Proof of Lemma 2. Fix a game with the discount factor $\delta \in (0,1)$ and the probability of market collapse $\varepsilon \in (0,1)$ in the similar way to the proof of Lemma 1 and fix an equilibrium in the required form. Let $q_g(\theta)$ be the unit of the asset a type-θ firm sells to the government before the market is open and $(q_1(\theta), q_2(\theta))$ be the units of the asset the firm sells in each of two periods, and $(t_g(\theta), t_1(\theta), \theta_2(\theta))$ be the corresponding transfers. The expected payoff for a type-θ firm when playing as if type-θ' is

$$\begin{aligned} u(\theta';\theta) = q_g(\theta')\big[[S + t_g(\theta')] + \varepsilon\delta\theta\big] + (1 - q_g(\theta'))\varepsilon(1+\delta)\theta \\ + (1-\varepsilon)\{1 - q_g(\theta')\}\big[q_1(\theta')\{S + t_1(\theta)\} + \{1 - q_1(\theta')\}\theta\big] \\ + (1-\varepsilon)\delta\big[q_2(\theta')\{S + t_2(\theta')\} + \{1 - q_2(\theta')\}\theta\big]. \end{aligned}$$

Let $Q(\cdot) = q_g(\cdot) + (1-\varepsilon)\{1 - q_g(\cdot)\}q_1(\cdot) + (1-\varepsilon)\delta q_2(\cdot)$, $T(\cdot) = q_g(\cdot)t_g(\cdot) + (1-\varepsilon)\{1 - q_g(\cdot)\}q_1(\cdot)t_1(\cdot) + (1-\varepsilon)\delta q_2(\cdot)t_2(\cdot)$. Since $u(\theta;\theta) - u(\theta';\theta) \geq 0$ and $u(\theta';\theta') - u(\theta;\theta') \geq 0$ for every $\theta \neq \theta'$ in equilibrium, it follows that $(S - \theta)[Q(\theta) - Q(\theta')] \geq T(\theta') - T(\theta)$ and $(S - \theta')[Q(\theta') - Q(\theta)] \geq T(\theta) - T(\theta')$. Combining these inequalities, we have

$$(\theta' - \theta)[Q(\theta) - Q(\theta')] \geq 0,$$

which implies that $Q(\theta)$ is decreasing in θ. Since $1 > (1-\varepsilon) > (1-\varepsilon)\delta$ and $q_j(\theta) \in \{0,1\}$ for every $j = g, 1, 2$ in pure-strategy equilibrium, there exist cutoffs $0 \leq \underline{\theta} \leq \tilde{\theta} \leq \hat{\theta} \leq \overline{\theta} \leq \breve{\theta} \leq 1$ such that *(i)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (1,0,1)$ if $\theta \leq \underline{\theta}$, *(ii)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,1,1)$ if $\theta \in (\underline{\theta}, \tilde{\theta}]$, *(iii)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (1,0,0)$ if $\theta \in (\tilde{\theta}, \hat{\theta}]$, *(iv)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,1,0)$ if $\theta \in (\hat{\theta}, \overline{\theta}]$, *(v)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,0,1)$ if $\theta \in (\overline{\theta}, \breve{\theta}]$, *(vi)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,0,0)$ if $\theta > \breve{\theta}$. Along with the D1 argument as in Step 3 in the proof of Lemma 1, we have $\hat{\theta} = \overline{\theta}$. This in turn means that any equilibrium must be characterized by the cutoffs $0 \leq \underline{\theta} \leq \tilde{\theta} \leq \hat{\theta} \leq \breve{\theta} \leq 1$. *Q.E.D.*

Proof of Proposition 4. To prove (i), we first characterize the range of p_g for which both BSS and BMS equilibria coexist. Fix any $S > \underline{S}_0$ and a bailout offer $p_g = \underline{p}_g^{BMS}$. This offer term induces a BMS equilibrium with the critical type $\theta_g = \theta_g^{BMS}$ such that $m(0, \theta_g^{BMS}) = I$ and $p_g = [\theta_g^{BMS} - m(0, \theta_g^{BMS}) - S] + m(\theta_g^{BMS}, \gamma(\theta_g^{BMS}))$. The log-concavity property of $f(\cdot)$ implies that $[\theta - m(0,\theta) - S] + m(\theta, \gamma(\theta))$ is increasing and continuous in θ, which guarantees uniqueness and existence of θ_g^{BMS}.

There are two possible cases, either $\underline{p}_g^{BMS} > p_0^*$ or $\underline{p}_g^{BMS} \leq p_0^*$. In the former case, take $p_g = \underline{p}_g^{BMS}$ and find the corresponding value $\theta_g' > 0$ which uniquely solves $p_g = m(\theta_g', \gamma(\theta_g'))$.

Since $\theta_g^{BMS} < \theta_0^*$, $m(0, \theta_g^{BMS}) = I$, and

$$p_g = m(\theta_g', \gamma(\theta_g')) = [\theta_g^{BMS} - m(0, \theta_g^{BMS}) - S] + m(\theta_g^{BMS}, \gamma(\theta_g^{BMS})),$$

we have $m(0, \theta_g') < I$. Thus the bailout term $\underline{p}_g^{BMS}$ also induces the BSS equilibrium with the cutoff $\theta_g = \theta_g^{BSS} \equiv \theta_g'$, which in turn means $\underline{p}_g^{BMS} < \overline{p}_g^{BSS} = p_0^*$. In the latter case, we have $\overline{p}_g^{BMS} = 1 \geq \overline{p}_g^{BSS}$ since the highest possible value of θ_g^{BMS} is 1. Consequently, both BSS and BMS equilibria exist for every $p_g \in [\underline{p}_g^{BMS} \vee p_0^*, \overline{p}_g^{BSS}) \neq \emptyset$.

To prove the first part of (ii), suppose $S > \underline{S}_0$ and fix any bailout term $p_g \in [\underline{p}_g^{BMS} \vee p_0^*, \overline{p}_g^{BSS})$. In the associated BSS equilibrium, the total volume of trade is $F(\gamma(\theta_g^{BSS}))$. In the BMS equilibrium corresponding to the same p_g, we have $\theta_g^{BMS} < \theta_0^*$ since $m(\theta_0^*, \gamma(\theta_0^*)) > m(\theta_g^{BSS}, \gamma(\theta_g^{BSS})) = p_g$. Since the firms $\theta \leq \theta_g^{BMS}$ sell their assets in every period and the firms $\theta \in (\theta_g^{BMS}, \gamma(\theta_g^{BMS}))$ sell only in $t = 2$ to the market, the total volume of trade is $F(\theta_g^{BMS}) + F(\gamma(\theta_g^{BMS}))$. Combined with $\theta_g^{BMS} > \theta_g^{BSS}$, we have $F(\theta_g^{BMS}) + F(\gamma(\theta_g^{BMS})) > F(\gamma(\theta_g^{BSS}))$.

To prove the second part of (ii), first suppose $S < \underline{S}^*$. We have $\theta_1^* = 0 < \theta_2^* = \theta_0^*$ in the equilibrium without bailout, so the total volume of trade in this equilibrium is $F(\theta_0^*)$. Since $\gamma(\theta_g^{BSS}) > \gamma(0) = \theta_0^*$, the BSS equilibrium induces more trade than without the bailout. Next, suppose $S \geq \underline{S}^*$, the cutoffs in the equilibrium without the bailout must satisfy $0 < \theta_1^* < \theta_2^* = \gamma(\theta_1^*)$ as well as $m(0, \theta_1^*) \geq I$. Since $m(0, \theta_g^{BSS}) < I$, we have $\theta_1^* > \theta_g^{BSS}$, and therefore, $F(\theta_g^{BSS}) < F(\theta_1^*) + F(\gamma(\theta_1^*))$. *Q.E.D.*

Proof of Proposition 5. We prove each part *(i)* - *(iii)* of Proposition 5 separately.

Proof of Proposition 5-(i). Consider $(\theta_g^{MR1}, \theta_1^{MR1})$ determined by

$$p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1) \text{ and } \theta_1 = \max\{\theta \geq \theta_g : \theta \leq 2m(\theta_g, \theta) - m(\theta, \gamma(\theta)) + S\}.$$

Assumption 2 guarantees existence and uniqueness of $(\theta_g^{MR1}, \theta_1^{MR1})$, as well as $\frac{d}{dp_g}\theta_g^{MR1} > 0$ for all bailout offers $p_g \geq 2m(0, \theta_1^*)$. This feature implies there exists $\underline{p}_g^{MR1}$ such that $m(0, \theta_g^{MR1}) \geq I$ if and only if $p_g \geq \underline{p}_g^{MR1}$. In addition, the log-concavity property of $f(\cdot)$ implies $\frac{\partial}{\partial a}m(a, b) + \frac{\partial}{\partial b}m(a, b) \leq 1$.[38] It follows that $\frac{d}{d\theta}\gamma(\theta) \leq 1$, and therefore, $\frac{d}{d\theta}(2m(\theta, \theta) - \theta - m(\theta, \gamma(\theta))) = \frac{d}{d\theta}(\theta - m(\theta, \gamma(\theta))) \geq 0$. Thus there exists $\hat{p}_g^{MR1}$ such that $\theta_g^{MR1} < \theta_1^{MR1}$ if and only if $p_g > \hat{p}_g^{MR1}$.

Next, consider $(\theta_g^{MR2}, \theta_1^{MR2}, \theta_{g\o}^{MR2})$ determined by

$$p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1), \quad m(0, \theta_g) + S = \theta_1, \text{ and } p_g = m(\theta_{g\o}, \gamma(\theta_{g\o})).$$

[38] See Szalay (2012) for more details.

Assumption 2 guarantees existence and uniqueness of those values, as well as $\frac{d}{dp_g}\theta_g^{MR2} > 0$ for all $p_g > 2m(0,S)$. This in turn means that there exists $\underline{p}_g^{MR2}$ such that $m(0,\theta_g^{MR2}) \geq I$ for all $p_g \geq \underline{p}_g^{MR2}$. Since $\frac{d}{d\theta}m(0,\theta) < 1$, there exists $\hat{p}_g^{MR2}$ such that $\theta_g^{MR2} < \theta_1^{MR2} = m(0,\theta_g^{MR2}) + S$ for all $p_g < \hat{p}_g^{MR2}$.

We first show that neither MR1 nor MR2 equilibrium exists for every $p_g \notin (\hat{p}_g^{MR1}, \hat{p}_g^{MR2})$. Consider $p_g \leq \hat{p}_g^{MR1} \wedge \hat{p}_g^{MR2}$. In this case, we have $\theta_g^{MR1} = \theta_1^{MR1}$ and $\theta_1^{MR2} > 2m(\theta_g^{MR2}, \theta_1^{MR2}) - m(\theta_1^{MR2}, \gamma(\theta_1^{MR2})) + S$. Thus, neither MR1 nor MR2 equilibrium can arise from this bailout offer. Next, consider any bailout offer $p_g \geq \hat{p}_g^{MR1} \vee \hat{p}_g^{MR2}$. Then we have $m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) < p_g$, thereby implying that MR1 equilibrium cannot arise because all firms $\theta \in (\theta_1^{MR1}, \gamma(\theta_1^{MR1})]$ would get a higher payoff by deviating and accepting the bailout in $t = 1$. Since $\theta_g^{MR2} > \theta_1^{MR2}$, MR2 equilibrium cannot arise from this bailout term.

We next show that either MR1 or MR2 equilibrium exists for every $p_g \in [\underline{p}_g^{MR1} \wedge \underline{p}_g^{MR2}, \infty) \cap (\hat{p}_g^{MR1}, \hat{p}_g^{MR2})$. Fix any $p_g \in [\underline{p}_g^{MR1}, \infty) \cap (\hat{p}_g^{MR1}, \hat{p}_g^{MR2})$. For this bailout term, we have either $m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) \geq p_g$ or $m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) < p_g$. In the former case, the corresponding cutoff values $(\theta_g^{MR1}, \theta_1^{MR1})$ satisfy $m(0, \theta_g^{MR1}) \geq I$ and $\theta_g^{MR1} < \theta_1^{MR1}$. Combined with $m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) \geq p_g$, these cutoff values satisfy the MR1 equilibrium conditions. In the latter case, the corresponding cutoff values $(\theta_g^{MR2}, \theta_1^{MR2}, \theta_{g\o}^{MR2})$ satisfy $\theta_g^{MR2} < \theta_1^{MR2} < \theta_{g\o}^{MR2}$ and $m(0, \theta_g^{MR2}) \geq I$, thereby satisfying the MR2 equilibrium conditions. Likewise, any $p_g \in [\underline{p}_g^{MR2}, \infty) \cap (\hat{p}_g^{MR1}, \hat{p}_g^{MR2})$ induces either MR1 or MR2 equilibrium. Consequently, we can find the range of bailout offers $[\underline{p}_g^{MR}, \overline{p}_g^{MR})$ in which every bailout offer p_g induces either MR1 or MR2 equilibrium, where $\underline{p}_g^{MR} = \min\{\underline{p}_g^{MR1}, \underline{p}_g^{MR2}, \hat{p}_g^{MR1}\}$ and $\overline{p}_g^{MR} = \hat{p}_g^{MR2}$.

We complete the proof by showing that every $p_g \in [\underline{p}_g^{MR}, \overline{p}_g^{MR})$ cannot be compatible with both MR1 and MR2 equilibria. Suppose to the contrary there is a p_g for which both MR1 and MR2 equilibria coexist. Since $m(\theta_{g\o}^{MR2}, \gamma(\theta_{g\o}^{MR2})) = p_g \leq m(\theta_1^{MR1}, \gamma(\theta_1^{MR1}))$, we must have $\theta_1^{MR2} < \theta_1^{MR1}$. Combining with this inequality, the indifference condition $p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1)$ in both equilibria implies $\theta_g^{MR2} > \theta_g^{MR1}$. However, it follows that

$$\theta_1^{MR1} \leq 2m(\theta_g^{MR1}, \theta_1^{MR1}) - m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) + S \leq (p_g - m(\theta_1^{MR1}, \gamma(\theta_1^{MR1}))) + m(0, \theta_g^{MR1}) + S,$$

whereas $\theta_1^{MR2} = m(0, \theta_g^{MR2}) + S$. This implies $\theta_1^{MR2} > \theta_1^{MR1}$, which is a a contradiction.

Proof of Proposition 5-(ii). Since $\theta_1^{MR1} > \theta_1^*$ following from $\theta_g^{MR1} > 0$, any MR1 equilibrium always induces larger total trade volume than the equilibrium without the bailout. To compare the total volume of trade in MR2 equilibrium to that without bailout, first consider $\theta_1^* \leq S$. In this case, we have $\theta_1^{MR2} > \theta_1^*$ following from $\theta_g^{MR2} > 0$ and $\theta_1^{MR2} = m(0, \theta_g^{MR2}) + S$. This in turn means that $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\o}^{MR2})) > F(\theta_1^*) + F(\gamma(\theta_1^*))$, so any MR2 equilibrium induces larger overall trade than without bailout. Next consider $\theta_1^* > S$. In this case, there exists $\tilde{\theta}_g > 0$ such that $\theta_1^* = m(0, \tilde{\theta}_g) + S$. Moreover, there exists $\tilde{p}_g^{MR}$ such that for every $p_g \leq \tilde{p}_g^{MR}$, there exist $(\theta_g^{MR2}, \theta_1^{MR2}, \theta_{g\emptyset}^{MR2})$ satisfying $\theta_g^{MR2} < \theta_1^{MR2} < \theta_{g\emptyset}^{MR2}$ and $\theta_g^{MR2} \leq \tilde{\theta}_g$. Since $\frac{d}{dp_g}\theta_g^{MR2}, \frac{d}{dp_g}\theta_1^{MR2}, \frac{d}{dp_g}\theta_{g\emptyset}^{MR2} > 0$, there exists $\breve{p}_g^{MR} \in [\underline{p}_g^{MR}, \tilde{p}_g^{MR}]$ such that for every $p_g \leq \breve{p}_g^{MR}$, there exists the MR2 equilibrium which yields the total volume of trade $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\emptyset}^{MR2})) \leq F(\theta_1^*) + F(\gamma(\theta_1^*))$. However, any MR2 equilibrium arising from some $p_g > \breve{p}_g^{MR}$ yields higher overall trading volume than the equilibrium without bailout. Proposition 5-(i) implies that no MR1 equilibrium arises from any $p_g \leq \breve{p}_g^{MR}$, thus the MR equilibrium yields lower overall trade if and only if $p_g \leq \breve{p}_g^{MR}$.

Proof of Proposition 5-(iii). We first show that $\theta_1 \leq \theta_0^*$ in both MR1 and MR2 equilibria. In MR1 equilibrium, we have $\theta_1 \leq m(0, \theta_g) + S$ because of indifference conditions $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$ and $\theta_1 + m(\theta_1, \gamma(\theta_1)) + S \leq 2m(\theta_g, \theta_1) + 2S$, as well as $p_g \leq m(\theta_1, \gamma(\theta_1))$ for all types $\theta \in (\theta_1, \gamma(\theta_1)]$ not to deviate. In MR2 equilibrium, we have $\theta_1 = m(0, \theta_g) + S$ by the corresponding indifference conditions on θ_g and θ_1. It thus follows that $\theta_1 \leq m(0, \theta_g) + S < m(0, \theta_1) + S$, which implies $\theta_1 \leq \theta_0^*$ where the equality holds only if $\theta_1 = \theta_0^* = 1$.

For some $p_g \in [\underline{p}_g^{MR}, \overline{p}_g^{MR})$, we may find the value of the critical type of the BMS equilibrium θ_g^{BMS}, which satisfies $p_g = m(\theta_g^{BMS}, \gamma(\theta_g^{BMS}))$ and $\theta_g^{BMS} > \theta_0^*$. In the corresponding BMS equilibrium,all types $\theta \leq \theta_g^{BMS}$ accept bailout in $t = 1$ while only those $\theta \leq \theta_0^*$ sell to the second-period market at the price $m(0, \theta_0^*)$. Thus the total volume of trade is $F(\theta_0^*) + F(\gamma(\theta_g^{BMS}))$, which is always greater than the total volume of trade in either MR1 or MR2 equilibrium since $\theta_1^{MR1} \vee \theta_1^{MR2} \leq \theta_0^* < \theta_g^{BMS}$.

Next consider a bailout offer $p_g \in [\underline{p}_g^{MR}, \overline{p}_g^{MR})$ which induces θ_g^{BMS}, as a unique solution to $p_g + m(0, \theta_g) + S = \theta_g + m(\theta_g, \gamma(\theta_g))$ subject to $\theta_g^{BMS} \leq \theta_0^*$. In the corresponding BMS equilibrium, all types $\theta \leq \theta_g^{BMS}$ sell their asset in every period and $\theta \in (\theta_g^{BMS}, \gamma(\theta_g^{BMS})]$ sell in $t = 2$ only, so the total volume of trade is $F(\theta_g^{BMS}) + F(\gamma(\theta_g^{BMS}))$. Suppose the same p_g induces an MR1 equilibrium. We show $\theta_g^{BMS} \geq \theta_1^{MR1}$ for this bailout offer, which implies the BMS equilibrium yields more trade than the MR1 equilibrium. If $\theta_g^{BMS} < \theta_1^{MR1}$, we have $\theta_1^{MR1} + m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) + S \leq p_g + m(0, \theta_g^{MR1}) + S$ from the indifference conditions on θ_1^{MR1}, which implies $\theta_g^{BMS} < \theta_g^{MR1}$. Log-concavity of the density $f(\cdot)$ implies $\frac{d}{d\theta}\big[\theta - m(0, \theta) + m(\theta, \gamma(\theta))\big] > 0$, thus we have $p_g + m(0, \theta_g^{MR1}) + 2S < \theta_g^{MR1} + m(\theta_g^{MR1}, \gamma(\theta_g^{MR1})) + S$. This inequality implies the cutoff type $\theta_g = \theta_g^{MR1}$ strictly prefers selling at the first-period market, a contradiction. Suppose next p_g induces an MR2 equilibrium. Since $\theta_g^{BMS} \leq \theta_0^*$ and $p_g + m(0, \theta_g^{BMS}) + 2S = \theta_g^{BMS} + m(\theta_g^{BMS}, \gamma(\theta_g^{BMS})) + S$, we have $p_g \leq m(\theta_g^{BMS}, \gamma(\theta_g^{BMS}))$. In this MR2 equilibrium, $p_g = m(\theta_{g\emptyset}^{MR2}, \gamma(\theta_{g\emptyset}^{MR2}))$, which implies $\theta_g^{BMS} \geq \theta_{g\emptyset}^{MR2} > \theta_1^{MR2} > \theta_g^{MR2}$. Since the total volume of trade in MR2 equilibrium is $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\o}^{MR2}))$, the feature $\theta_g^{BMS} \geq \theta_{g\o}^{MR2}$ implies that the BMS equilibrium induces more overall trade than the MR2 equilibrium. *Q.E.D.*

Proof of Lemma 3. Suppose to the contrary such an offer is refused by all firms in equilibrium. Then, the equilibrium described in Proposition 2 will obtain. Suppose any firm with sufficiently small θ deviates and accepts the secret bailout offer. There are two cases. Suppose first the equilibrium without government intervention involved an active market at price $p_1^* > 0$ in $t = 1$. Without deviating, such a firm will accept the market offer of p_1^* in $t = 1$, and sell at the same price in period $t = 2$, so the firm will enjoy the payoff of $2(p_1^* + S)$. By deviating, the firm will be regarded as an holdout (since no firm is supposed to accept bailout on the path) and will be offered the price of $p_2^* = m(\theta_1^*, \gamma(\theta_1^*)) > p_1^*$ in $t = 2$, so it will earn $p_g + p_2^* + 2S$. Since the latter profit exceeds the former, the deviation is strictly profitable. Suppose next the equilibrium without government intervention involved full market freeze. In this case, by not deviating the firm will earn θ in $t = 1$ and $p_0^* = m(0, \gamma(0))$ in $t = 2$. By deviating, the firm gets the same offer p_0^* in $t = 2$ while enjoying $p_g + S > \theta$ in $t = 1$. Hence, deviation is again strictly profitable. *Q.E.D.*

Proof of Lemma 4. Suppose to the contrary that there is an equilibrium with active market in $t = 1$ where a positive measure of firms sell their assets to the period-2 market only. As shown in Lemma 2, this equilibrium must be characterized by the cutoffs $0 < \underline{\theta} < \tilde{\theta} \leq \overline{\theta} \leq \hat{\theta} < \check{\theta} \leq 1$ such that: types $\theta \in [0, \underline{\theta}]$ sell in both periods with sales to the government in $t = 1$; $\theta \in (\underline{\theta}, \tilde{\theta}]$ sell in both periods with sales to the market in $t = 1$; $\theta \in (\tilde{\theta}, \overline{\theta}]$ sell only in $t = 1$ to the market; $\theta \in (\overline{\theta}, \hat{\theta}]$ sell only in $t = 1$ to the government; $\theta \in (\hat{\theta}, \check{\theta}]$ sell only in $t = 2$; and $\theta > \check{\theta}$ do not sell in any period. Let $\underline{p}$ be the equilibrium price at the period-1 market, $\overline{p}$ be the equilibrium price at the period-2 market offered to the firms without sales to the period-1 buyers, and $\tilde{p}$ be the equilibrium price offered to the firms with sales to the period-1 buyers, respectively. Then we have $\underline{p} \leq \tilde{p} \leq \overline{p}$ from the buyers' zero-profit conditions.

We first show that $\tilde{\theta} = \overline{\theta} = \hat{\theta}$. If $\tilde{\theta} < \overline{\theta} < \hat{\theta}$, we must have $p_g = \tilde{p} = \underline{p} = \overline{p}$. This inequality violates the necessary condition $\tilde{p} + S < \theta \leq \underline{p} + S$ for all $\theta \in (\tilde{\theta}, \overline{\theta}]$. If $\tilde{\theta} < \overline{\theta} = \hat{\theta}$, we must have $\underline{p} = m(\underline{\theta}, \overline{\theta})$ and $\tilde{p} = m(\underline{\theta}, \tilde{\theta})$ by the respective zero-profit conditions on the equilibrium prices, as well as $\overline{\theta} \leq m(\underline{\theta}, \overline{\theta}) + S$. However, the buyers in $t = 2$ could make a profit by offering $\tilde{\theta} + \varepsilon < \overline{p}$ for some $\varepsilon > 0$. If $\tilde{\theta} = \overline{\theta} < \hat{\theta}$, we must have $\tilde{p} = \underline{p} < p_g = \overline{p}$. With this inequality, the firms $\theta \in (\underline{\theta}, \tilde{\theta}]$ could get a higher payoff by deviating to sell to the government in $t = 1$, which is a contradiction.

It suffices to prove non-existence of the equilibrium supported by $0 < \underline{\theta} < \tilde{\theta} < \check{\theta} \leq 1$ under the secret bailout. To this end, we first define the following notations $\mathring{m}(a, b, c, d)$ and

$\mathring{\gamma}(a,b,c)$, where

$$\mathring{m}(a,b,c,d) := \mathbb{E}[\theta|\theta \in (a,b] \cup (c,d]]$$

and

$$\mathring{\gamma}(a,b,c) := \max\{\theta \geq c : \theta \leq \mathring{m}(a,b,c,\theta) + S\}.$$

Under the log-concavity assumption on $f(\cdot)$, we have that: *(i)* $\frac{\partial}{\partial d}\mathring{m}(a,b,c,d) < 1$; *(ii)* $\frac{\partial}{\partial b}\mathring{m}(a,b,c,d) < 0$ if and only if $b < \mathring{m}(a,b,c,d)$; *(iii)* $\mathring{\gamma}(a,b,c)$ is well-defined for every $0 \leq a \leq b \leq c \leq 1$.

In equilibrium, we have $\tilde{p} = \underline{p} = m(\underline{\theta}, \tilde{\theta})$ and $\overline{p} = \mathring{m}(0, \underline{\theta}, \tilde{\theta}, \check{\theta})$ which follow from the buyers' zero-profit conditions. Since $p_g + \overline{p} + 2S = 2\underline{p} + 2S = \tilde{\theta} + \overline{p} + S$ by the indifference condition on $\tilde{\theta}$, we have $\tilde{\theta} = p_g + S$. Furthermore, since $\check{\theta} = (\overline{p} + S) \vee 1$ by the indifference condition on $\check{\theta}$, we have $\check{\theta} = \mathring{\gamma}(0, \underline{\theta}, \tilde{\theta})$. For convenience, we henceforth abbreviate the notation $\mathring{\gamma}(0, \underline{\theta}, \tilde{\theta})$ to $\mathring{\gamma}(\underline{\theta})$.

For the cutoff type $\underline{\theta}$ to support this equilibrium, we must have

$$\mathring{m}(0, \underline{\theta}, \tilde{\theta}, \mathring{\gamma}(\underline{\theta})) > \underline{\theta}. \tag{19}$$

If not, we have $\mathring{m}(0, \underline{\theta}, \tilde{\theta}, \mathring{\gamma}(\underline{\theta})) \leq \underline{\theta} < m(\underline{\theta}, \tilde{\theta})$, which implies all types $\theta \in (\tilde{\theta}, \mathring{\gamma}(\underline{\theta})]$ would get a higher payoff by deviating and selling their asset at the period-1 market, a contradiction. Combining (19) with $\lim_{\theta \searrow \tilde{\theta}} \mathring{m}(0, \underline{\theta}, \tilde{\theta}, \theta) = m(0, \underline{\theta})$, $\frac{\partial}{\partial \theta}\mathring{m}(0, \underline{\theta}, \tilde{\theta}, \theta) < 1$, and $\tilde{\theta} = p_g + S$, we also have $p_g < m(0, \underline{\theta})$.

We next prove that the critical type $\underline{\theta}$ must satisfy the following:

$$\mathring{m}(0, \underline{\theta}, \tilde{\theta}, \mathring{\gamma}(\underline{\theta})) \leq \underline{\theta}, \tag{20}$$

which contradicts (19), and therefore, the supposed equilibrium cannot exist. To this end, define $\underline{\vartheta}$ which uniquely satisfies $m(0, \underline{\vartheta}) = p_g$. Following from the features $m(0, \underline{\vartheta}) < \underline{\vartheta}$ and $\lim_{\check{\theta} \searrow p_g + S} \mathring{m}(0, \underline{\vartheta}, \tilde{\theta}, \check{\theta}) + S = m(0, \underline{\vartheta}) + S = p_g + S$, there exists $\varepsilon > 0$ such that $\mathring{m}(0, \theta, \tilde{\theta}, \mathring{\gamma}(\theta)) < \theta$ for all $\theta \in (\underline{\vartheta}, \underline{\vartheta} + \varepsilon)$ by the log-concavity property of $f(\cdot)$ and continuity of $\mathring{m}(0, \theta, \tilde{\theta}, \mathring{\gamma}(\theta))$ in θ. Next, suppose there exists $\underline{\vartheta}' \in (\underline{\vartheta}, \tilde{\theta})$ such that $\underline{\vartheta}' \geq \underline{\vartheta} + \varepsilon$ and $\mathring{m}(0, \underline{\vartheta}', \tilde{\theta}, \mathring{\gamma}(\underline{\vartheta}')) = \underline{\vartheta}'$. Since *(i)* $\frac{\partial}{\partial d}\mathring{m}(a,b,c,d) < 1$ and *(ii)* $\frac{\partial}{\partial b}\mathring{m}(a,b,c,d) < 0 \iff b < \mathring{m}(a,b,c,d)$, we have $\frac{d}{d\theta}\left[\mathring{m}(0, \theta, \tilde{\theta}, \mathring{\gamma}(\theta)) - \theta\right]\big|_{\theta = \underline{\vartheta}'} < 0$. Since $\underline{\theta} \in (\underline{\vartheta}, \tilde{\theta})$, equation (20) must hold, which is the desired result. *Q.E.D.*

Proof of Proposition 7. Fix a bailout term p_g which induces an equilibrium with active market in $t = 1$. By Lemma 4, this equilibrium must be characterized by four cutoffs $0 < \theta_g < \theta_1 \leq$

$\theta_{g\o} \leq \overline{\theta}_1 \leq 1$, where the firms $\theta \in (\theta_{g\o}, \overline{\theta}_1]$ sell only in $t = 1$ to the market, while the other firms behave as in the SMR equilibrium.

We first show $\theta_{g\o} = \overline{\theta}_1$. In equilibrium, the buyers in $t = 2$ offer $m(\theta_g, \theta_1)$ to the firms selling to the market in $t = 1$ and $m(0, \theta_g)$ if not. Since the firms $\theta \in [0, \theta_1]$ must be indifferent between selling to the government and selling to the market in $t = 1$, we have $p_g + m(0, \theta_g) + 2S = p + m(\theta_g, \theta_1) + 2S$, where p is the equilibrium market price in $t = 1$. This in turn means that $p_g = p + m(\theta_g, \theta_1) - m(0, \theta_g) > p$. If $\theta_1 = \theta_{g\o} < \overline{\theta}_1$, we have $p = m(\theta_g, \overline{\theta}_1)$ and $\theta < p + S$ for all $\theta \in (\theta_g, \overline{\theta}_1)$. However, the log-concavity property of $f(\cdot)$ implies buyers in $t = 2$ can make a profit by offering $p' = m(\theta_g, \theta_1) + \varepsilon$ for some $\varepsilon > 0$. If $\theta_1 < \theta_{g\o} < \overline{\theta}_1$, all firms $\theta \in (\theta_1, \overline{\theta}_1]$ must get the same total payoff by selling to either the government or the market in $t = 1$, which in turn means $p_g + S + \theta = p + S + \theta \implies p_g = p$. However, this cannot be compatible with the indifference condition for the firms $\theta \in [0, 1]$. If $\theta_1 = \theta_{g\o} = \overline{\theta}_1 < 1$, we must have $\theta_1 = p + S < p_g + S$. However, the firms $\theta \in (\theta_1, p_g + S)$ can get a higher payoff by accepting the bailout in $t = 1$. Consequently, the SMR equilibrium must be shaped by three cutoffs $0 < \theta_g < \theta_1 \leq \theta_{g\o} \leq 1$, where $\theta_1 = \theta_{g\o}$ holds for $\theta_1 = 1$.

Next, consider the values $(\theta_g^{SMR}, \theta_1^{SMR}, \theta_{g\o}^{SMR})$ which solve

$$p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1), \;\; m(0, \theta_g) + S = \theta_1, \;\; \theta_{g\o} = (p_g + S) \wedge 1.$$

Assumption 2 ensures existence and uniqueness of those values for all $p_g > 2m(0, \theta_1^*)$, as well as $\frac{d}{dp_g}\theta_g^{SMR} > 0$.The latter feature implies that there exists $\underline{p}_g^{SMR}$ such that $m(0, \theta_g^{SMR}) \geq I$ for all $p_g \geq \underline{p}_g^{SMR}$. Since $\theta_1^{SMR} > \theta_g^{SMR}$ if and only if $\theta_g^{SMR} < m(0, \theta_g^{SMR}) + S$, there exists $\overline{p}_g^{SMR}$ such that $\theta_1^{SMR} > \theta_g^{SMR}$ if and only if $p_g < \overline{p}_g^{SMR}$. *Q.E.D.*

Proof of Proposition 8. In SNS equilibrium, firms $\theta \leq (p_g + S) \wedge 1$ accept the bailout in $t = 1$ and firms $\theta \leq \theta_0^*$ sell their assets to buyer in $t = 2$. Thus the total volume of trade is $F((p_g + S) \wedge 1) + F(\theta_0^*)$. In SMR equilibrium, firms $\theta \leq \theta_g^{SMR}$ accept the bailout in $t = 1$ and sell the remaining assets to the buyers in $t = 2$; firms $\theta \in (\theta_g^{SMR}, \theta_1^{SMR}]$ sell their assets to buyers in both periods; and firms $\theta \in (\theta_1^{SMR}, (p_g + S) \wedge 1]$ accept the bailout in $t = 1$ but do not sell their assets in $t = 2$. Thus the total volume of trade is $F((p_g + S) \wedge 1) + F(\theta_1^{SMR})$. Since $p_g + m(0, \theta_g^{SMR}) + 2S = 2m(\theta_g^{SMR}, \theta_1^{SMR}) + 2S$ and $\theta_1^{SMR} + p_g + S \leq 2m(\theta_g^{SMR}, \theta_1^{SMR}) + 2S$ in the SMR equilibrium, we have $\theta_1^{SMR} \leq m(0, \theta_g^{SMR}) + S$, and therefore, $\theta_1^{SMR} \leq \theta_0^*$. This in turn means that the SNS equilibrium yields more overall trade than the SMR equilibrium. *Q.E.D.*

Proof of Lemma 5. We begin with the proof of (i). The monotonicity of q follows from (IC), as is is standard (so we do not offer a proof). For the characterization of the second part, first define $\hat{\theta} := \sup\{\theta : q(\theta) = 2\}$ and $\check{\theta} := \sup\{\theta : q(\theta) = 1\}$. Also, define $\underline{t} := \inf_{\theta\in[0,1],\, i=1,2} t_i(\theta)$ and $\overline{t} := \sup_{\theta\in[0,1],\, i=1,2} t_i(\theta)$ for each period $i = 1, 2$. Note that $t_2(\theta) = \overline{t}$ for every $\theta \in (\hat{\theta}, \check{\theta}]$ and $\check{\theta} = \overline{t} + S$, or those high types would deviate and mimic the other types, which violates (IC).

We can characterize the second part by showing $\hat{\theta} \le \theta_0^* \le \check{\theta}$. We first show $\hat{\theta} \le \theta_0^*$. To do so, we consider the following two possible cases: either $t_2(\theta) = \overline{t}$ for some $\theta \le \hat{\theta}$ or $t_2(\theta) < \overline{t}$ for all $\theta \le \hat{\theta}$. Consider the former case and let $\hat{\Theta} \subset [0, \hat{\theta}]$ be the set of such types. Since $\overline{t} \ge t_2(\theta)$, we must have $t_1(\theta) = \underline{t}$ for all $\theta \in \hat{\Theta}$. Furthermore, (IC) implies that the firm $\hat{\theta}$ is indifferent between reporting its true type and mimicking as if the other types $\theta \le \hat{\theta}$:

$$t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S = \underline{t} + \overline{t} + 2S = \hat{\theta} + \overline{t} + S,$$

which implies $\hat{\theta} = \underline{t} + S$. If $\underline{t} \ge \mathbb{E}[\theta|\theta \in \hat{\Theta}]$, we would have $\hat{\theta} \ge \mathbb{E}[\theta|\theta \in \hat{\Theta}] + S$. However, the log-concavity property of $f(\cdot)$ implies $\partial_x \mathbb{E}[\theta|\theta \in \hat{\Theta} \cup (\hat{\theta}, x]] < 1$ for all $x > \hat{\theta}$. Combining this feature with $\lim_{x\searrow\hat{\theta}} \mathbb{E}[\theta|\theta \in \hat{\Theta} \cup (\hat{\theta}, x]] = \mathbb{E}[\theta|\theta \in \hat{\Theta}]$, we would have $\mathbb{E}[\theta|\theta \in \hat{\Theta} \cup (\hat{\theta}, \check{\theta}]] - \overline{t} < 0$. This means that the buyers in $t = 2$ must lose money by offering the transfer of $\overline{t}$ to the firm, thus we must have $\underline{t} < \mathbb{E}[\theta|\theta \in \hat{\Theta}]$. By construction of $\hat{\Theta}$, we also have

$$\overline{t} > \sup_{\theta\in[0,\hat{\theta}]\setminus\hat{\Theta},\, i\in\{1,2\}} t_i(\theta) \ge \inf_{\theta\in[0,\hat{\theta}]\setminus\hat{\Theta},\, i\in\{1,2\}} t_i(\theta) > \underline{t}.$$

The buyers' zero-profit condition implies $\underline{t} \le \mathbb{E}[t_2(\theta)|\theta \in [0, \hat{\theta}]\setminus\hat{\Theta}] = \mathbb{E}[\theta|\theta \in [0, \hat{\theta}]\setminus\hat{\Theta}]$. Combining this with $\underline{t} < \mathbb{E}[\theta|\theta \in \hat{\Theta}]$, we have $\underline{t} \le \mathbb{E}[\theta|\theta \le \hat{\theta}] = m(0, \hat{\theta})$, and therefore, $\hat{\theta} \le m(0, \hat{\theta}) + S$.

In the latter case, the firms $\theta \in (\hat{\theta}, \check{\theta}]$ must get a higher payoff by truthfully reporting than mimicking the others. Thus we have $\overline{t} \ge \sup_{\theta\in[0,\hat{\theta}]} t_i(\theta)$ for all $i = 1, 2$. The marginal type $\hat{\theta}$ must be indifferent between selling assets in both periods and only in one period at the highest price $\overline{t}$, i.e., $t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S = \hat{\theta} + \overline{t} + S$. Furthermore, the same type must be indifferent between reporting its true type and mimicking the other types $\theta \le \hat{\theta}$, i.e., $t_1(\hat{\theta}) + t_2(\hat{\theta}) = t_1(\theta) + t_2(\theta)$ for all $\theta \le \hat{\theta}$. Combining these two conditions, we have $\hat{\theta} \le t_2(\theta) + S$ for all $\theta \le \hat{\theta}$. Since the buyers' zero-profit condition, we have $\mathbb{E}[t_2(\theta)|\theta \le \hat{\theta}] = \mathbb{E}[\theta|\theta \le \hat{\theta}]$, and therefore,

$$\hat{\theta} \le \mathbb{E}[t_2(\theta)|\theta \le \hat{\theta}] + S = \mathbb{E}[\theta|\theta \le \hat{\theta}] + S \equiv m(0, \hat{\theta}) + S,$$

which implies the desired property.

We next show $\check{\theta} \ge \theta_0^*$. To show this, suppose to the contrary that $\check{\theta} < \theta_0^*$, or equivalently $\check{\theta} < m(0, \check{\theta}) + S$. Take $\check{\Theta} \subset (\hat{\theta}, \check{\theta}]$ be a set of types which receive the bailout in the first period but do not trade in the next period. Since $\check{\theta} = \overline{t} + S$ and the period-2 buyers must break even in equilibrium, we must have $\mathbb{E}[t_2(\theta)|\theta \notin \check{\Theta} \cap [0,\check{\theta}]] = \mathbb{E}[\theta|\theta \notin \check{\Theta} \cap [0,\check{\theta}]] < m(0,\check{\theta})$, thereby implying $\mathbb{E}[\theta|\theta \in \check{\Theta} \cap [0,\check{\theta}]] > m(0,\check{\theta})$. Therefore, there must exist a type $\theta' \in \check{\Theta}$ such that

$$\check{\theta} \geq \theta' > m(0,\check{\theta}),$$

which is a contradiction.

We next prove part (ii). First, recall

$$t(\theta) = u^M(\theta) - \theta(2 - q(\tilde{\theta})) - Sq(\tilde{\theta}). \tag{21}$$

Next, the envelope theorem applied to (IC) along with $u^M(1) = 2$ gives

$$u^M(\theta) = u^M(1) - \int_\theta^1 (2 - q(s))ds = 2 - \int_\theta^1 (2 - q(s))ds. \tag{22}$$

Substituting (21) and (22) into the welfare and integrating by parts gives:

$$\begin{aligned}
W(M) &= \int_0^1 \left[u^M(\theta) + (1+\lambda)\theta q(\theta) - (1+\lambda)t(\theta)\right] f(\theta)d\theta. \\
&= \int_0^1 \left[u^M(\theta) + (1+\lambda)\theta q(\theta) - (1+\lambda)(u^M(\theta) - \theta(2-q(\theta)) - Sq(\theta))\right] f(\theta)d\theta \\
&= \int_0^1 \left[(1+\lambda)Sq(\theta) - \lambda u^M(\theta) + 2(1+\lambda)\theta\right] f(\theta)d\theta \\
&= \int_0^1 \left[\left((1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}\right) q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta.
\end{aligned}$$

The expression in (15) is therefore derived. Revenue equivalence follows also from the observation that the welfare depends only on the allocation rule q.

We now prove part (iii). The welfare difference between the two equilibria is

$$\begin{aligned}
W_A - W_B &= \int_0^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&= \int_0^{\tilde{\theta}} J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta + \int_{\tilde{\theta}}^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&> \int_0^{\tilde{\theta}} J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)\theta + \int_{\tilde{\theta}}^1 J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&= J(\tilde{\theta}) \int_0^1 (q_A(\theta) - q_B(\theta))f(\theta)d\theta = 0,
\end{aligned}$$

where the inequality follows from the fact that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$ and that J is decreasing. The inequality must be strict if $q_A(\theta)$ and $q_B(\theta)$ differ on a positive measure of θ's, since J is strictly decreasing. *Q.E.D.*

Proof of Proposition 9. We prove each statement *(i)* - *(iii)* of Proposition 9 separately.

Proof of Proposition 9-(i). We show that either the MR1 equilibrium or the MR2 equilibrium is dominated in welfare by the BMS equilibrium under transparency.

First, suppose that a bailout term p_g induces an MR1 equilibrium under transparency. In this equilibrium, the firm sells $q_{MR1}(\theta) = 2$ units of the legacy asset for $\theta \leq \theta_1^{MR1}$, $q_{MR1}(\theta) = 1$ for $\theta \in (\theta_1^{MR1}, \gamma(\theta_1^{MR1})]$, and $q_{MR1}(\theta) = 0$ for $\theta > \gamma(\theta_1^{MR1})$. Because $\theta_1^{MR1} < \theta_0^* \equiv = \gamma(0)$, some bailout term p'_g can induce a BMS equilibrium with t=1 cutoff $\theta_g^{BMS} = \theta_1^{MR1}$. However, one cannot find any compatible BMS equilibrium in the boundary case ($\theta_g^{BMS} \neq \theta_{g\o}^{BMS}$) such that $\int_0^1 q_{MR1}(\theta)dF(\theta) = \int_0^1 q_{BMS}(\theta)dF(\theta)$. Had such a BMS equilibrium existed, it is required to have $\gamma(\theta_{g\o}^{BMS}) < \gamma(\theta_1^{MR1})$ due to $\theta_1^{MR1} < \theta_0^*$, but this implies $\theta_{g\o}^{BMS}) < \theta_1^{MR1} < \theta_0^* = \theta_g^{BMS}$, a contradiction to the feature $\theta_g^{BMS} \leq \theta_{g\o}^{BMS}$.

Second, consider an MR2 equilibrium. We claim that if there exists a BMS equilibrium with $\theta_g^{BMS} = \theta_{g\o}^{BMS}$ such that $\int_0^1 q_{MR2}(\theta)dF(\theta) = \int_0^1 q_{BMS}(\theta)dF(\theta)$, this BMS equilibrium strictly dominates in welfare the MR2 equilibrium. To show this, consider some bailout term p'_g that induces the BMS equilibrium with the t=2 cutoff $\gamma(\theta_g^{BMS}) = \gamma(\theta_{g\o}^{MR2})$. This BMS equilibrium generates a greater total trading volume than the MR2 equilibrium $\theta_g^{BMS} = \theta_{g\o}^{MR2} > \theta_1^{MR2}$. This feature implies that a BMS equilibrium with $\int_0^1 q_{MR2}(\theta)dF(\theta) = \int_0^1 q_{BMS}(\theta)dF(\theta)$ dominates the MR2 equilibrium in welfare because $q_{MR2}(\theta) \geq q_{BMS}(\theta)$ for $\theta \leq \theta_1^{MR2}$ and $q_{MR2}(\theta) \leq q_{BMS}(\theta)$ for $\theta > \theta_1^{MR2}$, where both inequalities are strict with a positive measure. In sum, any equilibrium with immediate market rejuvenation is dominated in welfare by the other type of equilibrium without immediate market rejuvenation under secrecy.

Proof of Proposition 9-(ii). We show that the SMR equilibrium is dominated in welfare by the SNS equilibrium under secrecy.

Fix a SMR equilibrium. A SNS equilibrium can arise from any $p_g \geq I$, so one can find a bailout term p'_g that can generate the same total trading volume as in the SMR equilibrium: $\int_0^1 q_{SNS}(\theta)dF(\theta) = \int_0^1 q_{SMR}(\theta)dF(\theta)$. Because $\theta_1^{SMR} < \theta_0^*$, there are two possible cases, either $\theta_g^{SNS} \equiv p_g + S \geq \theta_0^*$ or $\theta_g^{SNS} \equiv p_g + S < \theta_0^*$.

Consider the first case $\theta_g^{SNS} \equiv p_g + S \geq \theta_0^*$. By Lemma 5, the SNS equilibrium dominates the SMR equilibrium in welfare because $\theta_{g\o}^{SMR} > \theta_g^{SNS} > \theta_0^* > \theta_1^{MR}$. Next, consider the second case $\theta_g^{SNS} \equiv p_g + S < \theta_0^*$. In this case, we must have $\theta_g^{SNS} > \theta_1^{SMR}$ or else $\int_0^1 q_{SMR}(\theta)dF(\theta) >$

$\int_0^1 q_{SNS}(\theta)dF(\theta)$, a contradiction. As a result, we have $\theta_{g\o}^{SMR} > \theta_0^* > \theta_g^{SNS} > \theta_1^{MR}$, implying $q_{SMR}(\theta) \geq q_{SNS}(\theta)$ for $\theta \leq \theta_1^{SMR}$ and $q_{SMR}(\theta) \leq q_{SNS}(\theta)$ for $\theta \geq \theta_1^{MR}$ where the inequality strictly holds with a positive measure. By Lemma 5, the SNS equilibrium dominates the SMR equilibrium in welfare as well. In sum, any equilibrium without immediate market rejuvenation dominates the equilibrium with immediate market rejuvenation under secrecy.

Proof of Proposition 9-(iii). We show that any type of equilibrium under secrecy dominates the same type of equilibrium under transparency.

We compare the BMS equilibrium under transparency with the equilibrium without immediate market rejuvenation under secrecy. First, consider the BMS equilibrium under transparency with $\theta_g^{BMS} = \theta_{g\o}^{BMS}$. Since $\theta_g^{BMS} \leq \theta_0^* = \gamma(0)$ in this equilibrium, we can find a secret bailout program with a term p_g that induces a SNS equilibrium such that $\int_0^1 q_{SNS}(\theta)dF(\theta) = \int_0^1 q_{BMS}(\theta)dF(\theta)$. There are two possible cases of this SNS equilibrium, either $\theta_g^{SNS} \geq \theta_0^*$ or $\theta_g^{SNS} < \theta_0^*$. If $\theta_g^{SNS} \geq \theta_0^*$, then $\theta_g^{BMS} \leq \theta_0^*$ implies that it is required $\gamma(\theta_g^{BMS}) > \theta_g^{SNS} > \theta_0^* \geq \theta_g^{BMS}$ for the SNS equilibrium to generate the same total trading volume to that in the BMS equilibrium. For the same reason, $\theta_g^{SNS} > \theta_g^{BMS}$ if $\theta_g^{SNS} < \theta_0^*$. In both cases, Lemma 5 implies that the SNS equilibrium dominates the BMS equilibrium in welfare because $q_{SNS}(\theta) \leq q_{BMS}(\theta)$ for $\theta \leq \theta_g^{BMS}$ and $q_{SNS}(\theta) \geq q_{BMS}(\theta)$ for $\theta \geq \theta_g^{BMS}$, where the latter equality holds strictly with a positive measure. Second, consider another type of BMS equilibrium in which $\theta_g^{BMS} = \theta_0^* < \theta_{g\o}^{BMS}$. Then there always exists a SNS equilibrium that arises from a bailout term p'_g such that $\theta_g^{SNS} = \gamma(\theta_{g\o}^{BMS})$, therein $q_{SNS}(\theta) = q_{BMS}(\theta)$ for every $\theta \in [0, 1]$. In sum, any equilibrium without immediate market rejuvenation under transparency is dominated by the same type of equilibrium under secrecy.

Next, we compare the MR1 and MR2 equilibria under transparency with the SMR equilibrium under secrecy. First, suppose that a bailout term p_g and an MR1 equilibrium. The same offer p_g also induces the SMR equilibrium with the t=1 cutoff $\theta_1^{SMR} > \theta_1^{MR1}$ and the t=2 cutoff $\theta_{g\o}^{SMR} < \gamma(\theta_1^{MR1})$. Therefore, if there exists a SMR equilibrium such that $\int_0^1 q_{SMR}(\theta)dF(\theta) = \int_0^1 q_{MR1}dF(\theta)$, it is required $\theta_1^{SMR} > \theta_1^{MR1}$ and $\theta_{g\o}^{SMR} < \gamma(\theta_1^{MR1})$. Because $q_{MR1}(\theta) \geq q_{SMR}(\theta)$ for $\theta \leq \theta_1^{MR1}$ and $q_{MR1}(\theta) \leq q_{SMR}(\theta)$ for $\theta \geq \theta_1^{MR1}$ where the inequality strictly holds with a positive measure, the SMR equilibrium dominates the MR1 equilibrium in welfare by Lemma 5. If a bailout term p_g under transparency yields the MR2 equilibrium, the secret bailout with the same offer term yields the SMR equilibrium with $q_{SMR}(\theta) = q_{MR2}(\theta)$ for all $\theta \in [0, 1]$. Therefore, secrecy dominates transparency in welfare when both policies immediately rejuvenate the market in $t = 1$. In sum, secret bailouts dominate transparent bailouts whether or not both bailout policies rejuvenate the market in $t = 1$. *Q.E.D.*

Proof of Proposition 10. Let q be an arbitrary feasible allocation rule satisfying the constraints of $[P]$. Then,

$$
\begin{aligned}
& W(q^*) - W(q) \\
&= \int_0^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \\
&= \int_0^{\min\{\hat{\theta}^*, \theta_0^*\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta + \int_{\min\{\hat{\theta}^*, \theta_0^*\}}^{\max\{\hat{\theta}^*, \theta_0^*\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \\
&\quad + \int_{\max\{\hat{\theta}^*, \theta_0^*\}}^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta.
\end{aligned}
$$

The first integral is nonnegative since $J(\theta) \geq 0$ and $q(\theta) \leq 2 = q^*(\theta)$ for $\theta < \min\{\hat{\theta}^*, \theta_0^*\} \leq \hat{\theta}^*$. The last integral is also nonnegative since $J(\theta) \leq 0$ and $q(\theta) \geq 0 = q^*(\theta)$ for $\theta > \max\{\hat{\theta}^*, \theta_0^*\} \geq \hat{\theta}^*$. Finally, the non negativity of the middle integral can be seen as follows. Suppose first $\hat{\theta}^* < \theta_0^*$. Then, for any $\theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] = (\hat{\theta}^*, \theta_0^*]$, $J(\theta) \leq 0$ and $q(\theta) \geq 1 = q^*(\theta)$, so the middle integral is nonnegative. Suppose next $\hat{\theta}^* < \theta_0^*$. Then, for $\theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] = (\theta_0^*, \hat{\theta}^*]$, $J(\theta) \geq 0$ and $q(\theta) \leq 1 = q^*(\theta)$, so the middle integral is nonnegative. Since all three integrals are nonnegative, the allocation rule q^* is optimal.

The last statement follows from Proposition 6. *Q.E.D.*