Population Aging and Living Arrangement: Implication on Inequality

Sang-Hyop Lee
Andrew Mason
Hyun Kyung Kim

June 5, 2025

2025 KDI Journal of Economic Policy Conference
Population, Aging, and the Economy
Seoul, South Korea

Motivation

- Population aging raises inequality as within inequality rises by age (Deaton and Paxson, 1994, 1997)
- High income lowers the percentage of extended family (Costa, 1997)
  - This may in turn increase inequality, but rarely tested due to availability of data.
- Kim and Lee (2025)
  - Using the National Transfer Accounts (NTA) data of S. Korea which measures familial transfers comprehensively, they show that rapidly declining extended households can be more harmful than population aging in increasing inequality.
- Remaining question: does population aging also contributes to the declining extended households (in Korea or elsewhere).
  - Never been modeled nor tested.

1. Modeling 'Kinship' Structure (Basic set up: Mason and Lee 2004)

- Only two generations: *workers (w)* and *pensioners (p)*.
- The mortality of parents and offspring are independent
- No migration: only *f* (fertility rate) and *s* (survival rate).
- α: the proportion of *pensioners* who belong to a multi-generation family
- β: the proportion of *workers* who belong to a multi-generation family

→ Then, the population and the kinship structure is function of *s*, *f*, α, β

Also assume two-sex

- Introduce a two-sex pensioners: ϕ is correlation of survival between husband and wife
- $\beta = s + s(1-s)(1-\phi)$
- $\frac{\partial \beta}{\partial s} = 1 + (1-2s)(1-\phi)$
- $\eta_\beta = \frac{\partial \beta / \partial s}{\beta / s} = \frac{1+(1-2s)(1-\phi)}{1+(1-s)(1-\phi)}$
- If $0 \leq \phi \leq 1$, then an increase in the survival rate leads to an increase in the proportion of workers with surviving parents ($\partial \beta / \partial s \geq 0$) and $0 \leq \eta_\beta \leq 1$

By Workers' Age	Single family	Multi-generation	Entire Population
Worker's Generation	$(1-\beta)f$	$m_x^w = \frac{\beta f}{\alpha s + \beta f}$	$m^w = \frac{f}{s+f}$
Pensioner's Generation	$(1-\alpha)s$	$m_x^p = \frac{\alpha s}{\alpha s + \beta f}$	$m^p = \frac{s}{s+f}$
		$D_x^F = \frac{\alpha s}{\beta f}$	$D = \frac{s}{f} = \frac{m^p}{m^w}$

D: the dependency ratio of total population (*s/f*)

D_x^F: the dependency ratio of multi-generation families ($D_x^F = \frac{\alpha s}{\beta f} = \frac{\alpha}{\beta} D$)

$m_x^{P\,(w)}$: the share of pensioners (workers) in multi-generation families.

$m_x^F = m_x^w + m_x^p = \frac{\alpha s + \beta f}{s+f}$: the share of people in multi-generation families.

Comparative Statics (1)

Define $\alpha' \equiv \partial\alpha/\partial s$, $\beta' \equiv \partial\beta/\partial s$, $\alpha'' \equiv \partial\alpha/\partial f$, $\beta'' \equiv \partial\beta/\partial f$, $\eta_\alpha \equiv \frac{\partial\alpha}{\partial s}\frac{s}{\alpha}$, $\eta_\beta \equiv \frac{\partial\beta}{\partial s}\frac{s}{\beta}$

- Effects of population aging ($s\uparrow$, $f\downarrow$) on multi-generation family

$$\frac{\partial m_x^F}{\partial s} = \frac{(\alpha-\beta)f+\alpha\prime(s+f)s+\beta\prime(s+f)f}{(s+f)^2} \ (>0)$$

$$\frac{\partial m_x^F}{\partial f} = \frac{-(\alpha-\beta)s+\alpha\prime\prime(s+f)s}{(s+f)^2} \quad (<0)$$

$$\frac{\partial D_x^F}{\partial s} = \frac{\alpha}{\beta}\frac{1+\eta_\alpha-\eta_\beta}{f} \ (>0)$$

$$\frac{\partial D_x^F}{\partial f} = -\frac{\alpha s}{\beta f^2}(1-\eta'_\alpha) \ (<0)$$

Conclusion 1. Population aging is <u>more likely to</u> increase the share of pensioners or workers as well as dependency ratio in extended households.

2. Modeling Inequality

$V(Y)$: the variance in per adult household income for the population
$E(Y)$: is the mean income per adult
u_k:the proportion of the adult population belonging to the family cohort of age k
CV is the coefficient of variation

$$V(Y) = \sum_{k=a}^{a+g} u_k V(Y_k) + \sum_{k=a}^{a+g} u_k \{E(Y) - E(Y_k)\}^2$$

$$E(Y) = \sum_{k=a}^{a+g} u_k E(Y_k)$$

$$CV = \frac{\sqrt{V(Y)}}{E(Y)}$$

$$E(Y_k) = m_k^w E(Y_k^w) + m_k^p E\left(Y_k^p\right)$$

After thorough calculation,

$$V(Y_k) = w_1 V(Y_k^w) + \mathrm{w}_2 V\left(Y_k^p\right) + w_3 Cov\left(Y_k^w, Y_k^p\right) + w_4 \left\{E(Y_k^w) - E\left(Y_k^p\right)\right\}^2$$

$$w_1 = m^w - m_x^w m_x^p m_x^F$$

$$w_2 = m^p - m_x^w m_x^p m_x^F$$

$$w_3 = 2 m_x^w m_x^p m_x^F$$

$$w_4 = m^w m^p - m_x^w m_x^p m_x^F$$

Hence, $V(Y_k)$, is determined, in part, as a weighted average of

1) The variances of the incomes of the family members, $V(Y_k^w)$ and $V\left(Y_k^p\right)$
2) The covariance between their incomes, $C\left(Y_k^w, Y_k^p\right)$
3) The differences in the average incomes of workers and parents, $\left\{E(Y_k^w) - E\left(Y_k^p\right)\right\}^2$

Comparative Statics (2)

Define $R \equiv V(Y_k^p)/V(Y_k^w)$ and ρ as the correlation between the income of parents and workers,

$$\frac{\partial V(Y_k)}{\partial m^p} = V(Y_k^p) - V(Y_k^w) + (1 - 2m^p)\{E(Y_k^w) - E(Y_k^p)\}^2$$

$$\frac{\partial V(Y_k)}{\partial m_x^F} = m_x^w m_x^p \left[(2\rho\sqrt{R} - 1 - R) V(Y_k^w) - \{E(Y_k^w) - E(Y_k^p)\}^2 \right] \leq 0$$

$$(\because \; 2\rho\sqrt{R} - 1 - R = -(\rho - \sqrt{R})^2 - (1 - \rho^2) \leq 0)$$

$$\frac{\partial V(Y_k)}{\partial m_x^p} = (1 - 2m_x^p)_{m_x}^F \left[(2\rho\sqrt{R} - 1 - R) V(Y_k^w) - \{E(Y_k^w) - E(Y_k^p)\}^2 \right] \leq 0$$

as long as $m_x^p < 0.5$, which is always true.

Conclusion 2. The share of increase in pensioners or workers in extended households unambiguously decrease inequality.

3. Empirical Analysis

- Data: 7 years of micro level National Transfer Accounts (NTA) data (2010-2016)
  - Combine NTA with Korea's Household Income and Expenditure Survey (HIES) using age information of household members in the HIES and the information in the NTA to construct National Inclusion Accounts (NIA).

		Observation	Mean	Std. Dev.	Minimum	Maximum
(1)	D	210	0.669	0.582	0.119	2.915
(2)	D_x^F	210	0.935	0.356	0.135	2.473
(3)	m_x^F	210	0.227	0.093	0.120	0.462
(4)	m_x^P	210	0.465	0.103	0.119	0.712
(5)	Cohort	210	1972.5	8.905	1955	1990
(6)	$\ln labwork$	210	14.446	0.214	13.736	14.833
(7)	Incrat	210	0.309	0.381	0.000	1.489
(8)	Sexratio	210	0.399	0.069	0.128	0.539

Some controls: *Cohort* indicates the cohort effect; $\ln labwork$ indicates the natural log of workers' average earnings; *Incrat* represents the earnings of pensioners relative to workers; *Sexratio* indicates the sex ratio of the pensioner generation.

Results for proportion living in multi-generation households ($\ln m_x^F$)

	(1)	(2)	(3)	(4)
$\ln D$	0.084^{*} (0.043)	0.145^{**} (0.057)	0.448^{***} (0.074)	0.366^{***} (0.072)
$(\ln D)^2$		0.037 (0.023)		-0.168^{***} (0.032)
$\ln s$			-0.444^{***} (0.076)	-3.079^{***} (0.516)
$(\ln s)^2$				0.255^{***} (0.053)
Cohort	-0.001 (0.003)	0.002 (0.003)	0.016^{***} (0.004)	0.008^{*} (0.004)
$\ln labwork$	-1.182^{***} (0.099)	-1.186^{***} (0.098)	-1.098^{***} (0.092)	-0.833^{***} (0.098)
Incrat	0.215^{***} (0.082)	0.099 (0.109)	-0.264^{**} (0.112)	-0.021 (0.113)
Sexratio	-0.506^{***} (0.178)	-0.510^{***} (0.177)	-0.297^{*} (0.169)	-0.318^{*} (0.162)
Constant	17.072^{***} (5.753)	11.670^{*} (6.668)	-15.101^{**} (7.670)	5.339 (8.491)
N **(observation)**	210	210	210	210
Adj. R^2	0.8621	0.8631	0.8818	0.8975

Results for dependency ratio in multi-generation households ($\ln D_x^F$)

Variable	(1)	(2)	(3)	(4)
$\ln D$	1.132^{***} (0.058)	0.993^{***} (0.078)	0.770^{***} (0.105)	0.939^{***} (0.102)
$(\ln D)^2$		-0.085^{***} (0.032)		0.146^{***} (0.046)
$\ln s$			0.442^{***} (0.109)	4.590^{***} (0.734)
$(\ln s)^2$				-0.431^{***} (0.075)
Cohort	-0.001 (0.004)	-0.008^{*} (0.005)	-0.018^{***} (0.006)	-0.002 (0.006)
$\ln labwork$	0.781^{***} (0.135)	0.792^{***} (0.133)	0.698^{***} (0.132)	0.326^{**} (0.140)
Incrat	-0.685^{***} (0.112)	-0.423^{***} (0.148)	-0.209 (0.159)	-0.532^{***} (0.160)
Sexratio	-0.237 (0.244)	-0.226 (0.240)	-0.444^{*} (0.240)	-0.243 (0.231)
Constant	-7.455 (7.887)	4.823 (9.039)	24.548^{**} (10.937)	-12.892 (12.083)
N **(observation)**	210	210	210	210
Adjust R^2	0.8365	0.8414	0.8484	0.8691

$\ln D = \ln s - \ln f$, the elasticity of DxF with respect to s is equal to:
and the elasticity of DxF with respect to f is equal to:

Some calculation: make sense

$$\frac{\partial \ln m_x^F}{\partial \ln s} = (\beta_1 + \beta_3) + 2(\beta_2 + \beta_4)\ln s - 2\beta_2 \ln f$$
$$= -2.713 + 0.174 \ln s + 0.336 \ln f$$

$$\frac{\partial \ln m_x^F}{\partial \ln f} = -\beta_1 - 2\beta_2 \ln s + 2\beta_2 \ln f$$
$$= -0.366 + 0.336 \ln s - 0.336 \ln f$$

$$\frac{\partial \ln D_x^F}{\partial \ln s} = (\beta_1 + \beta_3) + 2(\beta_2 + \beta_4)\ln s - 2\beta_2 \ln f$$
$$= 5.529 - 0.570 \ln s - 0.292 \ln f$$

$$\frac{\partial \ln D_x^F}{\partial \ln f} = -\beta_1 - 2\beta_2 \ln s + 2\beta_2 \ln f$$
$$= -0.939 - 0.292 \ln s + 0.292 \ln f$$

Effect of living arrangements and population aging on income inequality (V: dependent)

Variable	(1)	(2)	(3)	(4)	(5)
m_x^F	-2.738*** (0.876)		-2.732*** (0.878)		
$\widehat{m_x^F}$				-0.234 (1.504)	-1.005 (1.571)
$\ln D$				0.088 (0.132)	0.403* (0.235)
m_x^P		-0.135 (0.401)	-0.105 (0.393)		-1.168 (0.720)
Cohort	-0.120*** (0.007)	-0.116*** (0.008)	-0.119*** (0.008)	-0.120*** (0.009)	-0.123*** (0.009)
$\ln labwork$	6.101*** (0.327)	6.690*** (0.294)	6.125*** (0.340)	6.590*** (0.426)	6.607*** (0.425)
Incrat	4.079*** (0.241)	3.660*** (0.205)	4.089*** (0.244)	3.600*** (0.309)	3.533*** (0.310)
Sexratio	-0.798 (0.512)	-0.441 (0.511)	-0.803 (0.513)	-0.456 (0.539)	-0.594 (0.543)
Constant	151.559*** (14.146)	133.711*** (15.717)	149.475*** (16.196)	144.449*** (20.720)	150.020*** (20.919)
N (observation)	210	210	210	210	210
Adjust R^2	0.8827	0.8770	0.8821	0.8766	0.8776

4. Summary

- Population aging could have had led to a greater increase in the proportion living in extended households
- Improvements in survival have a weaker effect than fertility decline on the proportion living in extended households.
- A rise in the earnings of pensioners relative to workers has no discernible effect on the proportion of family members living in extended households
- An increase in the proportion of the family cohort living in extended households reduces the variance in income. An increase in the proportion of pensioners in extended households reduces the variance in income
- Main limitation: the model implicitly assumes that income does not play a role in forming extended household. We tried, but wasn't an easy job.